31. Let $\{(X_i, \mathcal{T}_i) : i \in I\}$ be a family of topological spaces and $A_i \subseteq X_i$, for every $i \in I$. Show that

(1)
$$\prod \overline{A_i} = \overline{\prod A_i}.$$

(2) The set $\prod A_i$ is dense in $\prod X_i$ if and only if A_i is dense in X_i, for every $i \in I$.

(3) If $A_i \subseteq X_i$ and $A_j \subseteq X_j$, for some $i, j \in I$, then
$$A_i^o \times A_j^o = (A_i \times A_j)^o.$$

(4) It is *not* true in general that
$$\prod A_i^o = \left(\prod A_i\right)^o.$$

32. In the set of real numbers $\mathbb{R}$ we consider the equivalence relation "$\sim$" such that for $x \sim y$ if and and only if $x - y \in \mathbb{Z}$. Show that the identification space $\mathbb{R}/\sim$ is homeomorphic to $S^1 := \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$.

33. (**The orbit space**) Let X be a topological space and G be a group of homeomorphisms of X with composition as the group operation. Consider the relation "$\sim$" on X such that $x \sim y$ if and only if there exists $g \in G$ such that $y = g(x)$. Show that "$\sim$" is an equivalence relation on X and the canonical map $p : X \to X/\sim$ with $p(x) := G(x) = \{g(x) : g \in G\}$ is open.

34. Show that a topological space X is Hausdorff if and only if the *diagonal* $\Delta := \{(x, x) : x \in X\}$ is a closed subset of $X \times X$.

35. Let X, Y be two Hausdorff topological spaces and $f : X \to Y$ be a continuous map. Show that the *graph* $G_f := \{(x, f(x)) : x \in X\}$ of f is a closed subset of $X \times Y$.