36. Let X be a Hausdorff topological space and $A \subseteq X$. Show that

$$A = \bigcap \{V \subseteq X \,:\, V \text{ is open and } A \subseteq V\}.$$

37. Let X, Y be two topological spaces and $f : X \to Y$ be a continuous, open surjection. Show that Y is Hausdorff if and only if the set

$$\{\, (x_1, x_2) \,:\, x_1, x_2 \in X \text{ such that } f(x_1) = f(x_2) \,\}$$

is a closed subset of $X \times X$.

38. Let X be a metrizable space. Show that X is compact if and only if for every decreasing sequence $\{F_n \,:\, n \in \mathbb{N}\}$ (i.e. $F_{n+1} \subseteq F_n$ for every $n \in \mathbb{N}$) of non-empty closed subsets of X it holds that $\bigcap_{n=1}^{+\infty} F_n \neq \emptyset$.

39. Show that there is no continuous surjection $f : S^1 \to \mathbb{R}$.

40. Let (X, d) be a compact metric space and $f : X \to X$ be an *isometry* (i.e. for every $x, y \in X$ it holds that $d(f(x), f(y)) = d(x, y)$). Show that f is a surjection.