Group actions, horospheres, and simplices at infinity

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Abstract

Let G be a group that acts on a Hadamard manifold via covering space transformations. Suppose that G preserves horospheres at a finite collection of points $z_0, ..., z_k$ at infinity. One can ask the following questions:

1. Can one connect the points z_i and z_j by a path (and more generally, can one span $z_0, ..., z_k$ by a simplex) on which all horospheres are preserved by G?

2. If yes, then how nice (e.g. continuous, Holder, Lipschitz) can these paths (or simplices) be?

3. Let $Fix^0(G)$ be the set of points at infinity whose horospheres are preserved by G. What is the relation between the dimension of $Fix^0(G)$ and the dimension of G?

I will answer these questions in the case when the points z_i are mutually a distance at most $\pi/2$ apart. In short, the answer to the first question is yes, the answer to the second question is Lipschitz (we construct such simplices, called Busemann simlices), and the answer to the third is the following theorem.

Theorem: If the vertices $z_0, ..., z_k$ are at most $\pi/2$ apart and span a non-degenerate Busemann k-simplex, then the homological dimension of G is less than $n - k$.

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