Universität Bielefeld

SFB 701

Norddeutsches Gruppentheorie-Kolloquium

11–12 July 2014

Faculty of Mathematics
University of Bielefeld
Lecture Rooms: V2-205, V2-210/216

This workshop is part of the DFG-funded CRC 701
Spectral Structures and Topological Methods in Mathematics
at the University of Bielefeld

Organisers: Barbara Baumeister and Kai-Uwe Bux

`http://www.math.uni-bielefeld.de/~baumeist/ngk2014`

Schedule

Friday, July 11th, 2014
Lecture Room: **V2-205**

12:30 – 13:30	**Coffee** (Common Room V3-201)
13:30 – 14:15	**Bettina Eick** (Braunschweig) Cohomology and Quillen categories of finite p-groups
14:25 – 14:55	**Hung Tong-Viet** (Bielefeld) Recognising groups or group properties from character degrees of finite groups
15:05 – 15:50	**Gerhard Hiss** (Aachen) Tensor decomposable representations of finite groups
15:50 – 16:20	*Break*
16:20 – 17:00	**Benno Kuckuck** (Düsseldorf) Finiteness properties of fibre products of groups
17:10 – 17:40	**Imke Toborg** (Halle) A Z_3^*-type theorem viewed locally
17:50 – 18:20	**Anni Neumann** (Tübingen) Nilpotent subgroups of class ≤ 2 of maximal order
19:00	**Dinner**

Schedule

Saturday, July 12th, 2014
Lecture Room: **V2-210/216**

08:45 – 09:30	**Coffee** (Common Room V3-201)
09:30 – 10:15	**Richard Weiss** (Tufts) Groups Acting on Buildings
10:25 – 11:05	**Max Horn** (Giessen) Generalized Spin groups
11:05 – 11:35	*Break*
11:35 – 12:05	**Matthias Grüninger** (Louvain-la-Neuve) Trees, Twinnings and Moufang-sets
12:15 – 13:00	**Jan-Christoph Schlage-Puchta** (Rostock) The mixing time of the product replacement algorithm

Abstracts

Bettina Eick (Braunschweig)

Cohomology and Quillen categories of finite p-groups

The coclass of a finite p-group of order p^n and nilpotency class c is defined as $n-c$. For each coclass c and each prime p, there are infinitely many p-groups of this given coclass c. The classification and investigation of p-groups by coclass is an ongoing research project in p-group theory. As one step in this project the coclass families of p-groups have been defined. In this talk we exhibit a recent joint result with D. Green saying that 'almost all groups in a coclass family have equivalent Quillen categories'. This result has an impact on the mod-p cohomology rings of the considered groups and provides positive evidence for an open conjecture in p-group theory saying that 'there are only finitely many isomorphism types of mod-p cohomology rings for the infinitely many p-groups of fixed coclass'.

Hung Tong-Viet (Bielefeld)

Recognising groups or group properties from character degrees of finite groups

For a finite group G, let $\mathrm{X}_1(G)$ denote the complex irreducible character degree set of G counting multiplicity. Clearly, $\mathrm{X}_1(G)$ is just the first column of the character table of G and knowing $\mathrm{X}_1(G)$ is equivalent to knowing the complex group algebra $\mathbb{C}G$. Let $\mathrm{cd}(G)$ be the set obtained from $\mathrm{X}_1(G)$ by forgetting the multiplicity. It is well known that these two sets have a strong influence on the structure of the groups. In particular, the question that which groups or group properties can be determined by the character degrees has attracted much attention over the years. For example, a classical result of M.I. Isaacs in 1986 asserted that the nilpotency of a finite group G can be recognized from $\mathrm{X}_1(G)$. We also proved recently that every quasisimple group G is uniquely determined by $\mathrm{X}_1(G)$. In 2013, G. Navarro constructed an example showing that the set $\mathrm{cd}(G)$ does not determine the solvability of G. In this talk, I will discuss some related conjectures and counterexamples.

Gerhard Hiss (Aachen)

Tensor decomposable representations of finite groups

This talk is based on the diploma theses of Dorothee Ritter and Martin Couson, written under my direction.
Given two (matrix) representations of a finite group G over the same field, the Kronecker product of matrices yields a new representation of G, the tensor product of the given ones. A representation of G is tensor decomposable, if it is not equivalent to such a tensor product with factors of degree larger than 1.
Irreducible tensor decomposable representations are rare among the finite simple groups.
In contrast to this, finite p-groups appear to have many irreducible tensor decomposable representations over the complex numbers in general.
One of the main results of the above diploma theses is a characterization, in purely group theoretical terms, of those groups of order p^6 possessing an irreducible tensor decomposable representation over the complex numbers. This yields infinite series of such groups.

Benno Kuckuck (Düsseldorf)

Finiteness properties of fibre products of groups

Given a class of discrete groups with reasonably well-understood subgroups, such as the free groups or surface groups, what groups can occur as subgroups of direct products of these groups and how are these subgroups embedded inside the direct product? I will explain why higher finiteness properties play a crucial role in answering these questions. In particular, I will highlight a conjecture, the n-$(n+1)$-$(n+2)$-Conjecture, which formulates a criterion for a fibre products of groups to satisfy certain higher finiteness properties. We will see how this conjecture, in conjunction with celebrated work by Baumslag, Bridson, Howie, Miller and Short, would help to elucidate the subgroup structure of direct products of (for example) free groups. I will present a number of results towards proving the conjecture and describe their implications for the subgroup structure of direct products.

Imke Toborg (Halle)

A Z_3^*-type theorem viewed locally

I will present parts of my PhD-thesis that gives an alternative proof of Peter Rowley's theorem that 3-locally central elements of a finite simple group G are contained in $Z_3^*(G)$. The focus will be on the prime 3 and its specialities.

Anni Neumann (Tübingen)

Nilpotent subgroups of class ≤ 2 of maximal order

My talk is a part of my work about conjugacy of $\mathcal{N}$-maximal subgroups of a finite group G containing a nilpotent subgroup of class at most two of maximal order, known as BN-injectors. The notion of BN-injectors in arbitrary finite groups G is due to A. Bialostocki. Bialostocki conjectures that the BN-injectors are conjugate in any finite group G. The conjecture turns out to be false in general as a counterexample of P. Flavall shows. The aim now is to show that Bialostocki's conjecture is true for any finite group G apart from some special cases which can be enumerated. The first step in providing this assertion it is to show that the components of G are normalized by every $\mathcal{A} \in \mathcal{A}_2(G)$ (where $\mathcal{A}_2(G)$ is the set of all nilpotent subgroups of G of class at most two of maximal order) apart from some special cases which can be enumerated.
In my talk I will finally outline the proof that (except of some explicitly given cases) the BN-injectors of a group G whose components are of Chevalley-type, are conjugate.

Richard Weiss (Tufts)

Groups Acting on Buildings

Let Δ be a building and let Γ be a subgroup of its automorphism group. We denote by Δ^Γ the set of residues stabilized by Γ. We will describe a combinatorial version of the notion of a Tits index of a group acting on Δ which is valid for arbitrary buildings, we will discuss recent results giving necessary and sufficient conditions for Δ^Γ to be a building and necessary and sufficient conditions for Γ to have a Tits index, we will describe how these two results are related and we will mention some applications. This is joint work with Bernhard Mühlherr and Holger Petersson.

Max Horn (Giessen)

Generalized Spin groups

The classic spin groups $Spin(n)$ are double covers of the special orthogonal groups $SO(n)$ and play an important role in e.g. quantum physics. We generalize the concept of spin groups as follows: Observe that $SO(n)$ is the maximal compact subgroup of $SL(n,\mathbb{R})$, and also the centralizer of a Cartan-Chevalley involution of $SL(n)$. Given an arbitrary simply laced diagram Δ, one may define the simply-connected split real Kac-Moody group $G(\Delta)$ and its "maximal compact" subgroup $K(\Delta)$ as centralizer of a Cartan-Chevalley involution of $G(\Delta)$. Using local (rank 2) data and building theoretic methods, we then construct a covering group $Spin(\Delta)$ of $K(\Delta)$, and prove that it is indeed a double cover. For $\Delta = A_n$, this recovers precisely the classic setup. However, our approach works for arbitrary simply laced diagrams, such as E_{10}, and even for many other two-spherical diagrams. The E_{10} case in particular is of interest for string theory.

Matthias Grüninger (Louvain-la-Neuve)

Trees, Twinnings and Moufang-sets

Let T a locally finite tree, ∂T the set of ends of T and G a closed subgroup of $AutT$. We say that (T,G) induces a Moufang set at infinity if for all $\mathfrak{e} \in \mathrm{d}T$ there is a normal subgroup $U_\mathfrak{e}$ of $G_\mathfrak{e}$ which acts regularly on $\partial T \setminus \{\mathfrak{e}\}$. If K is a non-Archimedean localfield, G an algebraic group of rank 1 over K and T the Bruhat-Tits tree of G, then (T,G) is an example for this situation. P.-E. Caprace and T. De Medts could show that all examples with $U_\mathfrak{e}$ torsion-free arise from an algebraic group over a p-adic number field. In this talk we investigate the case that $U_\mathfrak{e}$ is an abelian group of exponent $p \in \mathbb{P}$. As a result we get that (apart from finitely many possible exceptions for $p \in \{2,3\}$ the group G is an algebraic group over a local field of charakteristic p. To gain this result we use that we can derive a RGD-system from G therefore can construct a twinning of T.

Jan-Christoph Schlage-Puchta (Rostock)

The mixing time of the product replacement algorithm

Let G be a d-generated finite group. Then we define a random walk on the set of generating d-tuples of G as follows: Pick two random indices $i \neq j$, then replace in the tuple $(g_1,\dots,g_d)$ the entry g_i by $g_i g_j$. Experimental evidence shows that the distribution of this random walk quickly converges to a limiting distribution, and that this limiting distribution is close to uniform. However, we cannot prove much for this walk. For example we do not know whether for $d=3$ and $G=S_n$ the underlying graph of this process is connected or not.
As d increases, the problem gets easier. Diaconis and Saloff-Coste showed that for G fixed and $d \to \infty$ the mixing time of the process is $\mathcal{O}(d^2 \log d)$ and conjectured, that the true order of the mixing time is $\mathcal{O}(d \log d)$. Chung and Graham proved this conjecture for $G=C_2$. Here we show that the conjecture is true, if we define mixing time with respect to total variation (i.e. L^1-norm). On the other hand the conjecture fails with respect to the L^p-norm for any $p > 1$. Here the mixing time is of order $\mathcal{O}(d^2)$.

Participant List

Fazli Amin	(Technische Universität Braunschweig)
Bernd Baumann	(Gießen)
Barbara Baumeister	(Bielefeld University)
Dung Duong	(Bielefeld University)
Bettina Eick	(TU Braunschweig)
Bernhard Fischer	(Bielefeld University)
Erik Friese	(Rostock)
Mathias Grimm	(Martin-Luther-Universität Halle-Wittenberg)
Matthias Grueninger	(Universite catholique de Louvain)
Johannes Hahn	(FSU Jena)
Hermann Heineken	(würzburg)
Ellen Henke	(University of Copenhagen)
Gerhard Hiss	(RWTH Aachen)
Max Horn	(JLU Gießen)
Linus Kramer	(WWU Münster)
Caroline Lassueur	(TU Kaiserslautern)
Peter Müller	(Universität Würzburg)
Kay Magaard	(University of Birmingham)
Gunter Malle	(TU Kaiserslautern)
Attila Maroti	(TU Kaiserslautern)
Marco Marschler	(Bielefeld University)
Anni Neumann	(Universität Tübingen)
Henning Niesdroy	(Bielefeld University)
Christian Pröseler	(MLU Halle-Wittenberg)
Kieran Roberts	(Bielefeld University)
Siddhartha Sarkar	(IISER Bhopal, India)
Jan-Christoph Schlage-Puchta	(Rostock)
Marcel Schmidt	(MLU Halle-Wittenberg)
Roland Schmidt	(Kiel)
Witzel Stefan	(Bielefeld University)
Bernd Stellmacher	(Kiel)
Gernot Stroth	(Halle)
Imke Toborg	(Koblenz Landau)
Hung Tong-Viet Phi	(Bielefeld University)
Christopher Voll	(Bielefeld University)
Rebecca Waldecker	(Halle)
Patrick Wegener	(Bielefeld University)
Richard Weiss	(Tufts)
Bettina Wilkens	(University of Botswana)
Chia Zargeh	(TU Braunschweig)