LECTURE 3

September 5, 2009

Locating $O_{p'}(G)$, locally

Given G - a minimal counterexample to CFSG - and A elementary abelian of rank ≥ 3, we can try to

- 1. identify $C_{O_{p'}(G)}(B)$ as a subgroup Θ_B of $O_{p'}(C_G(B))$, for each hyperplane $B < A$
- 2. prove that $\Theta_1 := \langle C_{O_{p'}(G)}(B) \,|\, |A:B| = p \rangle$ is a p'-group
- 3. prove that $\Theta_1 \triangleleft G$, or at least show that $N_G(\Theta_1)$ controls a great deal of p-local structure, e.g. contains a Sylow p-subgroup of G; ideally, is strongly p-embedded in G; prove that second alternative leads to a contradiction.
- 4. Conclude that $\Theta_B = 1$ for all $B < A$.

By doing this for appropriate Θ, aim for:
$C_G(z)$ resembles $C_{G^*}(z^*)$ for some known simple group G^*, $A^* \leq G^*$ elmentary abelian, and $z^* \in A^*$.
Aim for enough information to prove that $G \cong G^*$ or at least that G^* or some covering of G^* lies in G. Then uniqueness theorems $\implies G \cong G^*$.

Example 1. $p = 2$, G^* of Lie type, odd characteristic. Aiming for identification by Curtis-Tits. $O_{2'}(C_{G^*}(z^*))$ is cyclic for all involutions z^*. As a consequence,

$E\left[C_{G^*}(z^*)\right]O_{2'}(C_{G^*}(z^*))/O_{2'}(C_{G^*}(z^*)) =$

$E\left[(C_{G^*}(z^*)/O_{2'}(C_{G^*}(z^*))\right]$.

B-property for z^* in G^*.

Aim for B-property for z in G, for all $z \in A$ (A suitably chosen).

The Curtis-Tits generating subgroups will be subgroups of $E(C_G(z))$ for various z.

Example 2. $p = 2$, G^* of Lie type, characteristic 2. Then by Borel-Tits,

$$F^*(C_{G^*}(z^*)) = O_2(C_{G^*}(z^*)) \text{ for all involutions } z^*.$$

In particular $O_{2'}(C_{G^*}(z^*)) = 1$.

Aim for $O_{2'}(C_G(z)) = 1$ for all z, first for all $z \in A$.

The structure of $E(C_G(z))$ for various z determines which of these or other strategies to use.

G: Applications of signalizer functors - Balanced groups

The most pleasant application is to "balanced" groups. If A is an elementary abelian p-subgroup of G, then G is **balanced** with respect to A, by definition, if and only if there is an A-signalizer functor Θ on G such that $\Theta_{\langle a\rangle} = O_{p'}(C_G(a))$ for all $a \in A^{\#}$. (in other words, "$\Theta = O_{p'}$ is a signalizer functor.") For $B = \langle a, b\rangle \leq A$, this necessitates the "balance" condition $C_{\Theta_{\langle a\rangle}}(b) = C_{\Theta_{\langle b\rangle}}(a)$, and this condition is also sufficient. An extra advantage is that

$$\Theta^g_{\langle a\rangle} = \Theta_{\langle a^g\rangle} \text{ for all } a \in A^{\#},\ g \in G.$$

Theorem

Suppose that G is balanced with respect to A. Then $\Theta_1 = \langle \Theta_{\langle a\rangle} \mid a \in B^{\#}\rangle$ for any noncyclic $B \leq A$; consequently $N_G(B) \leq N_G(\Theta_1)$ for all noncyclic $B \leq A$. Moreover either

- *$O_{p'}(C_G(a)) = 1$ for all $a \in A^{\#}$; or*
- *$N := N_G(\Theta_1)$ is a proper subgroup of G containing not only $N_G(A)$ but also $N_G(B)$ for all noncyclic $B \leq A$.*

Gorenstein-Walter Alternative

Corollary (Gorenstein-Walter Alternative, instance 1 of many)

Suppose that G is simple and that every maximal elementary abelian 2-subgroup A of $S \in Syl_2(G)$ has order ≥ 8 and that G is balanced with respect to every such A. Then one of the following holds:

- (Sign. functor is trivial) $O_{2'}(C_G(z)) = 1 \ \forall$ *invol'n* $z \in G$.
- (Sign. functor is nontrivial) $\Gamma_{S,2}(G) := \langle N_G(T) \mid T \leq S,\ m_2(T) \geq 2 \rangle < G$.

Questions:

- How general is the hypothesis that G is balanced?
- How do we deal with the second alternative?

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Questions:

- How general is the hypothesis that G is balanced?
- How do we deal with the second alternative? Aschbacher did.

Corollary

Suppose that G is simple and that every maximal elementary abelian 2-subgroup A of $S \in Syl_2(G)$ has order ≥ 8 and that G is balanced with respect to every such A. Then $O_{2'}(C_G(z)) = 1$ for every involution $z \in G$.

Sidebar: Uniqueness Theorems

Aschbacher's theorem is an extension of the deep "strongly embedded subgroup" theorem, a combination of separate theorems of Bender ('71) and Suzuki ('64).

Theorem (Suzuki '64, Bender '71)

Suppose that $p = 2$, $P \in Syl_2(G)$, *and* M *is a subgroup of* G *such that*

- $P \leq M < G$
- $N_G(Q) \leq M$ *for all* $1 \neq Q \leq P$.

Then $O^{2'}(G/O_{2'}(G)) \cong L_2(2^n)$, $Sz(2^n)$, $U_3(2^n)$ *("Bender groups") or* $m_2(G) = 1$.

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Most uniqueness theorems are concerned with one of the objects defined as follows.

H: Uniqueness Theorems

Let P be a p-subgroup of G. Let k be a positive integer such that $k < m_p(P)$. Then

$$\Gamma_{P,k}(G) = \left\langle N_G(Q) \,|\, Q \leq P,\ m_p(Q) \geq k \right\rangle$$

$$\Gamma'_{P,k}(G) = \left\langle C_G(Q) \,|\, Q \leq P,\ m_p(Q) \geq k \right\rangle$$

For P elementary abelian,

$$\Gamma'_{P,-k}(G) = \Gamma'_{P,m_p(P)-k}(G)$$

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If $P \in \mathrm{Syl}_p(G)$, then $\Gamma_{P,k}(G)$ and its conjugates are the minimal subgroups $\Gamma \leq G$ with the properties

$$P \leq \Gamma \text{ and } m_p(\Gamma \cap \Gamma^x) < k \text{ for all } x \in G - P.$$

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A **uniqueness theorem** is a theorem whose hypothesis includes $\Gamma_{P,k}(G) < G$ for some $k < m_p(G)$ and $P \in \mathrm{Syl}_p(G)$ and whose conclusion either gets an improved version of this (smaller k, or $p = 2$ instead of original p, or a classification).

Sample uniqueness theorems

Here are several examples (always $P \in \mathrm{Syl}_p(G)$):

Theorem (Suzuki '64, Bender '71)

If $p = 2$, $\Gamma_{P,1}(G) < G$, then $O^{2'}(G/O_{2'}(G))$ is a "Bender group" $L_2(2^n)$, $Sz(2^n)$, $U_3(2^n)$, or $m_2(G) = 1$.

Theorem (Aschbacher '74)

If $p = 2$ and $\Gamma_{P,2}(G) < G$, then $O^{2'}(G/O_{2'}(G))$ is known.

Theorem (Aschbacher-Gorenstein-Lyons '81)

Suppose that

- *$p > 2$ and $\Gamma_{P,3}(G) \leq M < G$,*
- *whenever T is a 2-subgroup of M such that $m_p(N_M(T)) \geq 4$, then $N_G(T) \leq M$.*

Then the condition $m_p(N_M(T)) \geq 4$ can be weakened to $m_p(N_M(T)) \geq 3$.

This can in turn be used to prove $\Gamma_{P,2}(G) \leq M$.

Theorem (Aschbacher '83)

Suppose that

- $F^*(N) = O_2(N)$ *for all 2-local subgroups* $N \leq G$
- *for any maximal 2-local subgroup* $M < G$*, any prime* $p \in \sigma(M) := \{p \mid m_p(M) \geq 4\}$*, and* $P \in Syl_p(M)$*, we have* $\Gamma_{P,1}(G) \leq M$
- $\sigma(M) \neq \emptyset$ *for some maximal 2-local subgroup* M
- *for any 2-local subgroup* N *such that* $m_p(N) \geq 2$*, we have* $N \leq M$*.*

Then G *has a strongly embedded subgroup for the prime 2 (* $\Longrightarrow$ *contradiction).*

This "Uniqueness Case" theorem in the first generation CFSG had a precursor in Thompson's work on N-groups. It illustrates that the case of groups G of characteristic 2-type involved significant p-local analysis for certain odd primes p.

I: Local vs global balance

We have defined G to be (globally) balanced with respect to the elementary abelian p-group A iff $O_{p'}(C_G(a)) \cap C_G(b) = O_{p'}(C_G(b)) \cap C_G(a)$ for all $a, b \in A^{\#}$. The following fundamental result enables one to spot the obstruction to this condition.

Theorem (Part of $L^*_{2'}$-balance, Gorenstein-Walter)

Let X be a finite group and a an involution acting on X. Then

$$O_{2'}(C_X(a)) \leq O_{2',E,2'}(X).$$

This gets applied to $X = C_G(b)$:

$$O_{2'}(C_G(a)) \cap C_G(b) \leq O_{2'}(C_{C_G(b)}(a)) = O_{2'}(C_X(a))$$

and if $O_{2'}(C_X(a)) \leq O_{2'}(X) = O_{2'}(C_G(b))$, we have one inclusion of the two symmetric conditions needed for the balance condition.

Theorem

Continuing, $O_{2'}(C_X(a)) \leq O_{2'}(X)$ iff there is no component $\overline{L}$ of $E(\overline{X})$, $\overline{X} = X/O_{2'}(X)$, such that $O_{2'}(C_X(a))$ normalizes and acts nontrivially on $\overline{L}$. Such a component is said to be **locally unbalancing** *with respect to $\langle a \rangle$.*

When this occurs, $\langle a \rangle$ normalizes $\overline{L}$ and

$$O_{2'}(C_{\mathrm{Aut}(\overline{L})}(a)) \text{ is nontrivial}^1.$$

We say that $\overline{L}$ is **locally unbalancing**. Thus local balance of all components of $X/O_{2'}(X) = C_G(b)/O_{2'}(C_G(b))$, $b \in A^{\#}$, implies the "global" balance of G with respect to A.

[1] Statement exaggerated a bit.

Example 2 bis. If $F^*(C_G(b)/O_{2'}(C_G(b))) = O_2(C_G(b)/O_{2'}(C_G(b))$ for all involutions $b \in A$, then G is balanced with respect to A.

Corollary

Suppose that G is simple and that every maximal elementary abelian 2-subgroup A of $S \in Syl_2(G)$ has order ≥ 8 and that $E(C_G(b)/O_{2'}(C_G(b))) = 1$ for all involutions $b \in G$. Then $O_{2'}(C_G(z)) = 1$ for every involution $z \in G$.

To avoid such obstructions, one can define nontrivial signalizer functors not using $O_{2'}(C_G(a))$, but a more complicated construction based on subgroups

$$\Delta_G(D) = \bigcap_{d \in D^{\#}} O_{2'}(C_G(d))$$

for various (noncyclic) $D \leq A$, and beyond that based on $[\Delta_G(D), A]$ as well. These subgroups are less likely than $O_{2'}(C_G(a))$ to act nontrivially on $\overline{L}$.
For example we say that G is 3/2-balanced with respect to A iff

$$[A, O_{2'}(C_G(a)) \cap C_G(b)] \leq O_{2'}(C_G(b)) \text{ and}$$

$$\Delta_G(D) \cap C_G(b) \leq O_{2'}(C_G(b))$$

for all $a, b \in A^{\#}$ and for all $D \leq A$, $D \cong Z_2 \times Z_2$

We say that a quasisimple $\mathcal{K}$-group $\overline{L}$ is locally 3/2-balanced with respect to the elementary abelian 2-subgroup $\overline{A} \leq \mathrm{Aut}(\overline{L})$ essentially if and only if $\Delta_{\mathrm{Aut}(\overline{L})}(\overline{D}) = [O_{2'}(C_{\mathrm{Aut}(\overline{L})}(\overline{b})), \overline{A}] = 1$ for all $\overline{D} \leq \overline{A}$ of rank at most 2 and all $\overline{b} \in \overline{A}^{\#}$.

$3/2$-balance for groups of Lie type is a natural condition from the algebraic group point of view; the counterexamples can be pinpointed.

Lemma

Suppose that $L = \mathbf{L}^F$ is a group of Lie type. Let p be a prime different from the underlying characteristic, and let A be a noncyclic elementary abelian p-subgroup of L. If A is "simply abelian in L" - its preimage in the universal version $\widetilde{L}$ of L is abelian – then L is $3/2$-balanced with respect to A.

This is a consequence of the fact that any rank 2 abelian p-subgroup of $\mathbf{L}$ lies in a torus
(Steinberg connectedness + every s.s. element in a connected group lies in a torus)

Finally, the above discussion applies almost without change to any prime p (and $O_{p'}$ instead of $O_{2'}$). The change is that

$L^*_{p'}$-balance ($p > 2$) is a **Theorem**, not a Theorem.

The stars would go if one could make the following "localization" of the Schreier Conjecture a Theorem. Glauberman did just that, but only for $p = 2$.

Theorem (Glauberman, $p = 2$)
Observation ($p > 2$)

Let G be a finite simple group and $P \in Syl_p(G)$. Then $C_{Aut(G)}(P)$ is p-nilpotent, i.e. has a normal p-complement.

J: Amalgams

Exploiting the condition $O_p(G) = 1$ for all primes p dividing $|G|$.

Fix a prime p (most likely $p = 2$ but not necessarily) and let G be a group of characteristic p-type in the sense that $m_p(G) > 1$ and all p-local subgroups N containing a Sylow p-subgroup of G satisfy

$$F^*(N) = O_p(N).$$

If $O_p(\langle N_1, N_2 \rangle) \neq 1$ for all p-local subgroups N_1, N_2 containing a fixed $P \in \mathrm{Syl}_p(G)$, then P lies in a unique maximal p-local subgroup of G. This case aside, N_1 and N_2 may be chosen so that

$$O_p(\langle N_1, N_2 \rangle) = 1,$$

giving the classic amalgam setup. The choice of N_1 and N_2 may be tailored to the situation at hand. Often one of the N_i's is a minimal parabolic, meaning that P is contained in a unique maximal subgroup of that N_i.

MSS

The **Meierfrankenfeld-Stellmacher-Stroth project** (MSS '03) ("3rd-generation proof") is devoted to the analysis of G via such amalgams. Central is the idea that the condition $O_p(\langle N_1, N_2\rangle) = 1$ imposes stringent conditions on the N_i-modules contained in $O_p(N_i)$, conditions related to failure of factorization, low degree minimal polynomials, and low numbers of chief factors. With extra hypotheses on N_1 and N_2, these conditions may permit the identification of the "amalgam"

$$N_1 \leftarrow N_1 \cap N_2 \rightarrow N_2.$$

The MSS project aims toward a classification of amalgams (and groups) arising in simple groups of characteristic p-type.

Borel-Tits Theorem $\implies$ all groups of Lie type over field of characteristic p are of characteristic p-type or of p-rank 1.

K: Decomposability: Component Theorems

We referred above to $L^*_{2'}$-balance and $L^*_{p'}$-balance. The simpler notion of $L_{2'}$-balance is fundamental to the calculus of components of local subgroups. The p-layer $L_{p'}(G)$ is:

$$L_{p'}(G) = O^{p'}(O_{p',E}(G)).$$

The factorization $E(G/O_{p'}(G)) = E_1 \cdots E_n$ into components leads to a factorization $L_{p'}(G) = L_1 \cdots L_n$ of "p-components", sometimes perversely called "p'-components."

Theorem

If N is a p-local subgroup of G, then $L_{p'}(N) \leq L_{p'}(G)$. Equivalently $L_{p'}(N_G(P)) = L_{p'}(N_{L_{p'}(G)}(P))$.

For $p = 2$ it's a Theorem.

Aschbacher component theorem

Theorem

Assume that $L_{2'}(N) = E(N)$ *for all 2-locals* N *of the group* G *of even order, and that* $E(C_G(t))$ *has a component of 2-rank 2 for some involution* $t \in G$*. Then*

- *There exists an involution* t *and a component* K *of* $E(C_G(t))$ *such that* $m_2(K) \geq 2$ *and* K *is a component of* $E(C_G(u))$ *for all involutions* $u \in C_G(K)$*. (terminology:* K *is "terminal".)*
- *If* G *is simple, then for any such* t *and* K*,* $C_G(K)$ *contains no* G*-conjugate of* K*.*

This was generalized in GLS:

Theorem (GLS 4)

Suppose that $m_p(G) \geq 4$, M is a maximal subgroup of G, and K is a p-component of M. Let P and Q be Sylow p-subgroups of M and $C_M(K/O_{p'}(K))$ respectively, with $Q \leq P$. Assume that

- $m_p(Q) \geq 2$
- $\Gamma'_{Q,1}(G) \leq M$

Then either M is strongly p-embedded in G or there exists $U \leq Q$, $U \cong Z_p \times Z_p$, and $g \in G - M$ such that $U^g \leq M$. Moreover $K \lhd M$.

The subgroup M arises in GLS typically as $N_G(\Theta_1)$ for some nontrivial signalizer functor defined on subgroups of $(P \cap K)Q$.

K: Decomposability: strongly closed p-subgroups

Theorem (Goldschmidt '75)

Let $T \in Syl_2(G)$. Let S be a strongly closed 2-subgroup of T. (For any $x \in S$, $x^G \cap T \subseteq S$.) Then

$$C_G(S)^{(\infty)} \lhd G.$$

If $T = S_1 \times S_2$ with each S_i strongly closed in T, and $O_{2'}(G) = 1$, then $[\langle S_1^G \rangle, \langle S_2^G \rangle] = 1$.

(Counterexamples to $S \in \mathrm{Syl}_2(\langle S^G \rangle)$: $G = Sz(2^n)$, $U_3(2^n)$.)

GLS Report

Reference: A.M.S. Surveys and Monographs **40**. Nr's 1-6, ...

- Use a well-defined set of well-accepted references.
- Base case analysis on local structure of a minimal counterexample G
- Case division based on p-ranks of subgroups and mainly on the sets of groups

$$\mathcal{L}_p(G) = \{K \,\big|\, K \text{ is a component of } E(C/O_{p'}(C)),$$
$$C = C_G(x),\, x \in G,\, x^p = 1 \neq x\}$$

$$\mathcal{L}_p(G) \subseteq \mathcal{K}_p := \{K,\ \text{known nonabel. quasisimple group } \big|\, O_{p'}(K) = 1\}$$

For each p, define a tripartition

$$\mathcal{K}_p = \mathcal{C}_p \cup \mathcal{T}_p \cup \mathcal{G}_p$$

Seen from a distance (or even close-up if $p > 11$)

$$Chev(p) \cup \{K \mid m_p(K) = 1\} \cup \{\text{everything else}\}$$

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