

Title: Factorizations of the lattice of reflection subgroups of a Coxeter group

Abstract: The notion of an orthogonal factorization system generalizes the canonical surjective-injective factorization of functions in the category of sets. We specialize standard properties of such factorization systems to the context of complete lattices, and apply them to the study of reflection subgroups of Coxeter groups. Well-known classes of reflection subgroups, such as standard parabolic, parabolic and locally parabolic subgroups, are associated to natural factorizations of the lattice of reflection subgroups. We also discuss some interesting new classes of reflection subgroups within this framework. Some consequences of the general theory of factorization systems in the context of Coxeter groups have not been previously observed, and the theory provides a useful conceptual framework which systematizes many of the facts, provides more uniform proofs and suggests new concepts and questions.

Reading list for the Mini-Course:

Coxeter systems, standard parabolic subgroups, parabolic subgroups

- Humphreys, Introduction to Coxeter groups and reflection groups [Chapter 5]
- Bourbaki, Lie groups and Lie algebras, Ch IV-VI. [Chapter IV]

Reflection subgroups of a Coxeter system:

- Dyer, Hecke algebras and reflections in Coxeter groups, Ph.D. Thesis, (1987), University of Sydney. [Chapter 3]. Available at <https://www3.nd.edu/~dyer/>
- Deodhar, A note on subgroups generated by reflections in Coxeter groups, Arch. Math, 53 (1989), 543-546
- Dyer, Reflection Subgroups of Coxeter systems, Journal of Algebra,(1990) 135, 57-73 [Sections 2-4]

Locally parabolic subgroups:

- Nuida, Locally parabolic subgroups in Coxeter groups of arbitrary rank, J. Alg. 350 (2012), 207-217.

Dual Coxeter systems:

- Dyer, On minimal lengths of expressions of Coxeter group elements as products of reflections, Proc. A.M.S., 129 (2001), 2591-2595
- Baumeister, Dyer, Stump and Wegener, A note on the transitive Hurwitz action on decompositions of parabolic Coxeter elements, Proc. A.M.S., Series B, 1 (2014), 149-154.

Orthogonal factorization systems (a quick introduction for those with suitable category-theoretic background, which will not be required for the workshop)

- Riehl, Factorization systems: Section 1, available at <https://math.jhu.edu/~eriehl/>