

Title: Complex reflection, braid groups, and Hecke algebras

Abstract: Let W be a finite complex reflection group. In our course we introduce two fundamental structures associated to W : the braid group and the generic Hecke algebra. In the case of Weyl groups, these play a central role in the representation theory of finite groups of Lie type. We will discuss presentations of these objects in terms of braid diagrams, generalising the Coxeter presentations. We will further highlight the now proven freeness conjecture and the existence of trace forms. We intend, if time permits, to end on some recent developments on associated Yokonuma type algebras.

Some relevant literature:

1) On reflection groups:

- [1] M. Geck, G. Malle, Reflection groups. Pp. 337–383 in: *Handbook of Algebra, Vol. 4*, Elsevier, 2006.

This survey article collects some properties of finite reflection groups.

2) On braid groups:

- [2] M. Broué, G. Malle, R. Rouquier, Complex reflection groups, braid groups, Hecke algebras. *J. reine angew. Math.* **500** (1998), 127–190.
[3] D. Bessis, Zariski theorems and diagrams for braid groups. *Invent. Math.* **145** (2001), 487–507.

The article [2] introduces the braid groups for arbitrary complex reflection groups and uses these to construct Hecke algebras; [3] contains a general proof of their presentations.

3) On Hecke algebras:

- [4] M. Broué, G. Malle, Zyklotomische Heckealgebren. *Astérisque* **212** (1993), 119–189.
[5] M. Broué, G. Malle, J. Michel, Towards spetses I. *Transform. Groups* **4** (1999), 157–218.
[6] M. Geck and G. Pfeiffer, *Characters of Finite Coxeter Groups and Iwahori-Hecke Algebras*. The Clarendon Press, Oxford University Press, New York, 2000.
[7] P. Etingof, Proof of the Broué–Malle–Rouquier conjecture in characteristic zero (after I. Losev and I. Marin–G. Pfeiffer). *Arnold Math. J.* **3** (2017), 445–449.

The article [4] introduces the generic Hecke algebras for arbitrary complex reflection groups; [5] discusses some properties and formulates the freeness and symmetrising trace conjectures; [6] is the standard reference for the case of Coxeter groups; [7] is a survey on the recent proof of the freeness conjecture.