Title: Interval monoids

Abstract: Many of most commonly studied Garside monoids are interval monoids.

An interval monoid is constructed from an interval of a group. If S is a generating set for the group G, this defines a length on G: for $g \in G, l(g)$ is the minimum number of elements of S of which g is the product. It defines also two partial orders: for $a, b \in G$ we say that a left divides b if $l(b) = l(a) + l(a^{-1}b)$. Right divisibility is defined symmetrically. Given $c \in G$, such that the set of its left and right divisors coincide (we say that c is balanced) we construct a monoid with generators in bijection with the set D of divisors of c, and a relation $ab = c$ whenever $a, b, c \in D$ satisfy $l(c) = l(a) + l(b)$, called the interval monoid defined by the interval D.

The main theorem in the theory of interval monoids is that if D is a lattice for right and left divisibility, then the interval monoid is Garside.

I will explain the above theorem and give examples.

As examples, both the ordinary and dual braid monoids for finite Coxeter groups are interval monoids. The construction of the dual braid monoid can be extended to well-generated finite complex reflections groups (in the non-well generated case, it is possible to construct an "interval Garsoide category"). Some other monoids turn out to be interval. For example, Georges Neaime has show that such is the case of the Corran-Picantin monoid.

There will be a computer exercise session with examples using the Garside functions in Chevie.