Title: Artin groups and related spaces

Abstract: Let A be a group and let $S \subset A$ be a generating set of A. We say that (A, S) is an Artin system if A admits a presentation with S as set of generators and with relations of the form $\underbrace{sts\cdots}_{m} = \underbrace{tst\cdots}_{m}$. In that case A is called an Artin group. If W is the quotient of A by the relations $s^2 = 1$, $s \in S$, then W is a Coxeter group and (W, S) is a Coxeter system. The flagship example of an Artin group is the braid group $\mathcal{B}_n$ on n strands which admits the following presentation:

$$\mathcal{B}_n = \langle s_1, \dots, s_{n-1} \mid s_i s_j = s_j s_i \text{ for } |i-j| \ge 2\,,\ s_i s_j s_i = s_j s_i s_j \text{ for } |i-j| = 1 \rangle\,.$$

The quotient of $\mathcal{B}_n$ by the relations $s_i^2 = 1$, $1 \le i \le n-1$, is the symmetric group $\mathfrak{S}_n$.

The natural epimorphism $A \to W$ which sends s to s for all $s \in S$ has an interpretation in terms of covering maps which, in the case of the braid group $\mathcal{B}_n$, can be presented as follows. Let

$$M_n = \{(z_1, \dots, z_n) \in \mathbb{C}^n \mid z_i \ne z_j \text{ for } i \ne j\}\,.$$

The symmetric group $\mathfrak{S}_n$ acts on M_n by permutation of the coordinates, and this action is free. The quotient $N_n = M_n/\mathfrak{S}_n$ is the space of configurations of n distinct points in $\mathbb{C}$, and its fundamental group is $\mathcal{B}_n$. Furthermore, N_n is a classifying space for $\mathcal{B}_n$, which means among other things that the homology of $\mathcal{B}_n$ is the same as that of N_n. The extension of this situation to all Artin groups will be the topic of the mini-course.

Although the mini-course will be as self-contained as possible, a background on algebraic topology and on the theory of Coxeter groups may be useful. Regarding algebraic topology, I recommend the following book:

- A. Hatcher. Algebraic topology. Cambridge University Press, Cambridge, 2002.

For Coxeter groups, I recommend the following books:

- M. W. Davis. The geometry and topology of Coxeter groups. London Mathematical Society Monographs Series, 32. Princeton University Press, Princeton, NJ, 2008.
- J. E. Humphreys. Reflection groups and Coxeter groups. Cambridge Studies in Advanced Mathematics, 29. Cambridge University Press, Cambridge, 1990.

There is no book on Artin groups, but introductions to the theory that deal especially with the topic of the mini-course can be found in the following references.

- J. McCammond. The mysterious geometry of Artin groups. Winter Braids Lect. Notes 4 (2017), Winter Braids VII (Caen, 2017), Exp. No. 1, 30 pp.
- L. Paris. $K(\pi, 1)$ conjecture for Artin groups. Ann. Fac. Sci. Toulouse Math. (6) 23 (2014), no. 2, 361–415.
- L. Paris. Lectures on Artin groups and the $K(\pi, 1)$ conjecture. Groups of exceptional type, Coxeter groups and related geometries, 239–257, Springer Proc. Math. Stat., 82, Springer, New Delhi, 2014.