DOBRUSHIN'S COMPACTNESS CRITERION FOR EUCLIDEAN GIBBS MEASURES

TATIANA PASUREK

Abstract. We prove existence, uniqueness, and uniform à-priori estimates for Euclidean Gibbs measures corresponding to certain quantum systems with unbounded spins, pair potentials of superquadratic growth, and infinite radius of interaction. We use Dobrushin's criteria and give a direct construction of appropriate compact functions on loop spaces.

1. Introduction

The paper is concerned with interacting systems of ν-dimensional quantum anharmonic oscillators, living on some countable set $\mathbb{L} \subset \mathbb{R}^d$. In Statistical Physics, the object of our study is well-known as *quantum anharmonic crystals.* However, we essentially extend a class of models under consideration, e.g., by allowing for the interaction to be *non-translation invariant* and to have *infinite range* and *superquadratic growth*, so that the most of previous results do not apply in this case.

A mathematical description of equilibrium properties of quantum systems can be given in terms of their Gibbs states defined on proper algebras of observables (cf. [11]). We follow here the *Euclidean* (or *path space*) *approach*, which remains so far the only method which allows to construct and study Gibbs states for infinite systems of quantum particles described by unbounded operators. This approach was first implemented to quantum lattice systems in [1]; for further developments see [2], [4], [8], [18], [19], [21]. Briefly speaking, we transform the problem of giving a proper meaning to a quantum Gibbs state ρ_β into the problem of studying a certain *Euclidean Gibbs measure* μ on the *'temperature loop lattice'* $\Omega := [C(S_\beta)]^{\mathbb{L}}$ (cf. Section 4 below for rigorous definitions). Here $\beta := 1/T > 0$ is the inverse (absolute) temperature, and the '*spin space'* $C(S_\beta)$ consists of all continuous functions (loops) defined on the circle $S_\beta \cong [0, \beta]$. But, as compared with classical lattice systems, the situation with Euclidean Gibbs measures is much more complicated, since now the spin (i.e., loop) spaces themselves are infinite dimensional and their topological features should be taken into account carefully. Also, as is typical for non-compact spin spaces, we have to restrict ourselves to the set $\mathcal{G}^{\mathrm{t}}$ of *tempered* Gibbs measures, which we specify by some natural support condition (cf. (4.10) below).

The aim of this paper is to establish an *elementary new method* for proving *existence, uniqueness and à-priori estimate* for the tempered Euclidean Gibbs measures. It is based on the famous Dobrushin compactness condition (cf. (6.2) below)

1991 *Mathematics Subject Classification.* Primary 82B10; secondary 60H30.

Key words and phrases. Dobrushin–Lanford–Ruelle equations, Gibbs measures, temperature loop spaces, Dobrushin criteria .

Supported by the Lise-Meitner Habilitation Stipendium and by the DFG through SFB 701 Bielefeld.

and seems to be overlooked before. Moreover, this approach is quite universal for lattice models (classical and quantum) and gives structural insight. It allows us to get improvements of essentially all related existence results, in particularly those obtained by superstability estimates, cluster expansions, reflection positivity, integration by parts, and stochastic quantization (see, e.g., [1], [4], [7], [10], [23], [25], [26]). We note that the method straightforwardly extends to the many-particle interactions of superquadratic growth, unbounded order, and infinite range, as well as to the spin systems defined on graphs.

The organization of this paper is as follows. In Section 3 we introduce the models of quantum lattice systems. In Section 4 we recall details on the corresponding Gibbsian formalism for Euclidean Gibbs measures μ on the loop lattice Ω. In Section 5 we formulate our main Theorems 5.1, 5.2, and 5.6 respectively on the existence, à-priori estimates, and uniqueness for tempered Euclidean Gibbs measures $\mu \in \mathcal{G}^{\mathrm{t}}$, as well as compare our results with those previously obtained by other authors. In Section 6 we outline the basic ideas of the proofs. In Section 7 we discuss some possible generalizations of our method.

Finally we mention the related publications [20], [21] focused on the quantum systems with harmonic pair interactions and the manuscript [24] dealing in more detail with the classical spin systems.

2. The Model

We consider an infinite system of interacting quantum particles performing ν-dimensional anharmonic oscillations around their equilibrium positions which form a countable set $\mathbb{L} \subset \mathbb{R}^d$. Such system is described by the heuristic Hamiltonian

$$H = -\frac{1}{2m}\sum_{\ell} |p_l|^2 + \frac{a}{2}\sum_{\ell} |q_\ell|^2 + \frac{1}{2}\sum_{\ell,\ell'} W_{\ell\ell'}(q_\ell, q_{\ell'}) + \sum_{\ell} V_\ell(q_\ell), \tag{2.1}$$

where $m > 0$ is the particle's mass and $a > 0$ is their rigidity. To each particle indexed by ℓ, there correspond the canonical displacement and momentum operators, q_ℓ and $p_\ell = -i\mathrm{d}/\mathrm{d}q_\ell$, acting in the physical state space $\mathcal{H}_\ell = L^2(\mathbb{R}^\nu, \mathrm{d}q_\ell)$. Note that $\nu, d \in \mathbb{N}$ may be arbitrary and do not need to coincide. We use the standard notation $(\cdot,\cdot)$ and $|\cdot|$ for the scalar product and distance in all Euclidean spaces ($\mathbb{R}^d$, $\mathbb{R}^\nu$ etc.). The sums $\sum_\ell$ and $\sum_{\ell,\ell'}$ are running respectively over all $\ell \in \mathbb{L}$ and ordered pairs $(\ell,\ell') \in \mathbb{L}^2$. For a set $\Lambda \subset \mathbb{L}$, by $|\Lambda|$ we denote its cardinality and by Λ^c the complement. We write $\Lambda \Subset \mathbb{L}$ if Λ is non-void and finite. As usual, $\Lambda \nearrow \mathbb{L}$ means the limit taken along any increasing sequence of volumes $\Lambda^{(N)} \subseteq \Lambda^{(N+1)} \Subset \mathbb{L}$ such that $\bigcup_{N\in\mathbb{N}} \Lambda^{(N)} = \mathbb{L}$. Moreover, we impose the following condition of spatial regularity

$$\sup_{\ell} \sum_{\ell'} (1 + |\ell - \ell'|)^{-(d+\epsilon)} < \infty, \qquad \text{for all } \epsilon > 0, \tag{2.2}$$

which surely holds if $\mathbb{L}$ is the integer lattice $\mathbb{Z}^d$. In the later case the model is called a quantum anharmonic crystal.

The interaction potentials are given by continuous functions $V_\ell : \mathbb{R}^\nu \to \mathbb{R}$, $W_{\ell\ell'} = W_{\ell'\ell} : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$, satisfying the following conditions:

Assumption (W)**:** There exist $R \geq 2$ and a symmetric matrix $\mathbf{J} = (J_{\ell\ell'})_{\mathbb{L}\times\mathbb{L}}$ with the nonnegative entries and zero diagonal, such that

$$|W_{\ell\ell'}(q,q')| \leq \frac{J_{\ell\ell'}}{2}(1+|q|^R+|q'|^R), \quad \text{for all } \ q,q' \in \mathbb{R}^\nu. \tag{2.3}$$

Assumption (J)**:** The matrix $\mathbf{J}$ is fastly decreasing, that is,

$$||\mathbf{J}||_p \overset{\text{def}}{=} \sup_\ell \sum_{\ell'} J_{\ell\ell'}(1+|\ell-\ell'|)^p < \infty, \quad \text{for all.} p \geq 0. \tag{2.4}$$

Assumption (V)**:** There exist a continuous function $V : \mathbb{R}^\nu \to \mathbb{R}$ and constants $P > R$, $A_V > 0$, and $B_V \in \mathbb{R}$, such that for all $\ell \in \mathbb{L}$ and $q \in \mathbb{R}^\nu$

$$A_V|q|^P + B_V \leq V_\ell(q) \leq V(q). \tag{2.5}$$

Remark 2.1. Typical examples are the polynomials

$$V_\ell(q) := \sum\nolimits_{\mathrm{s}=1}^{\mathrm{p}} b_\ell^{(\mathrm{s})}|q|^{2\mathrm{s}}, \quad W_{\ell\ell'}(q,q') := \sum\nolimits_{\mathrm{s}=1}^{\mathrm{r}} c_{\ell\ell'}^{(\mathrm{s})}|q-q'|^{2\mathrm{s}},$$

in which $1 \leq \mathrm{p} < \mathrm{r}$ and the coefficient $b_\ell^{(s)}$ and $c_{\ell\ell'}^{(s)}$ are uniformly kept in certain intervals. In fact, from V_ℓ one always can extract a quadratic term $U(q_\ell) := a|q_\ell|^2/2 + (h, q_\ell)$ with any $a > 0$ and $h \in \mathbb{R}^\nu$, so that (2.5) is still true for the potentials $\tilde{V}_\ell := V_\ell - U$ with arbitrary $\tilde{A}_V < A_V$. Merely speaking, our conditions mean that the inter-particle interaction is dominated by the self-potentials, which implies a lattice stabilization. The case $P = R$ is allowed as well, but it needs a more accurate analysis (cf. Subsection 7.1 below).

Lattice systems of such type are commonly viewed in quantum statistical physics as mathematical models of a crystalline substance (for more physical background see, e.g., [1, 2, 15, 17]). A complete description of thermal equilibrium properties of quantum systems might be given in terms of their Gibbs states. As was already mentioned in Introduction, we take the Euclidean (i.e., path space) approach first implemented to quantum lattice systems by S. Albeverio and R. Hoegh-Krohn in [1]. In Sections 3 and 4 we proceed with the rigorous description of the corresponding Gibbsian formalism (for a detailed exposition see, e.g., [4], [21]).

3. Loop Spaces

Euclidean Gibbs measures μ associated with the quantum system (2.1) at the inverse temperature $\beta = 1/T > 0$ are defined on the spaces of β-periodic functions (or *loops*, for short). Let $S_\beta \cong [0,\beta]$ be a circle with Lebesgue measure $\mathrm{d}\tau$ and distance$|\tau-\tau'|_\beta := \min\{|\tau-\tau'|;\beta-|\tau-\tau'|\}$, $\tau,\tau' \in S_\beta$. As *single spin spaces* at each $\ell \in \mathbb{L}$, we shall use the standard Banach spaces

$$\begin{gathered} L_\beta^r := \ L^r(S_\beta \to \mathbb{R}^\nu, \mathrm{d}\tau), \quad r \geq 1, \\ C_\beta := C(S_\beta \to \mathbb{R}^\nu), \quad C_\beta^\sigma := C^\sigma(S_\beta \to \mathbb{R}^\nu), \quad \sigma \in (0,1/2), \end{gathered} \tag{3.1}$$

of all integrable resp. (Hölder-) continuous functions $\omega_\ell : S_\beta \to \mathbb{R}^\nu$ with the norms

$$\begin{gathered} |\omega_\ell|_{L_\beta^p} := \left[\int_{S_\beta} |\omega_\ell(\tau)|^p \mathrm{d}\tau\right]^{1/p}, \\ |\omega_\ell|_{C_\beta} := \sup_{\tau\in S_\beta} |\omega_\ell(\tau)|, \quad |\omega_\ell|_{C_\beta^\sigma} := |\omega_\ell|_{C_\beta} + \sup_{\tau,\tau'\in S_\beta,\, \tau\neq\tau'} \frac{|\omega_\ell(\tau)-\omega_\ell(\tau')|}{|\tau-\tau'|_\beta^\sigma}. \end{gathered} \tag{3.2}$$

One has the dense continuous embeddings $C_\beta^\sigma \hookrightarrow C_\beta \hookrightarrow L_\beta^p$ with the following relation between the corresponding Borel σ-algebras: $C_\beta^\sigma \in \mathcal{B}(C_\beta) = \mathcal{B}(L_\beta^p) \cap C_\beta$.

Given $\Lambda \subseteq \mathbb{L}$, we define the spaces of configurations

$$\Omega_\Lambda := \{\omega_\Lambda = (\omega_\ell)_{\ell\in\Lambda} \mid \omega_\ell \in C_\beta\}, \quad \Omega = \Omega_{\mathbb{L}} := \{\omega = (\omega_\ell)_{\ell\in\mathbb{L}} \mid \omega_\ell \in C_\beta\}. \tag{3.3}$$

Each Ω_Λ is a Polish space equipped with the product topology and with the corresponding Borel σ-algebra $\mathcal{B}(\Omega_\Lambda)$. In particular, Ω is the *configuration space* for the whole system. By $\mathcal{P}(\Omega)$ we denote the set of all probability measures on $(\Omega, \mathcal{B}(\Omega))$.

To control the support properties of measures $\mu \in \mathcal{P}(\Omega)$, we introduce certain classes of reasonable configurations $\omega \in \Omega$. Their choice is strongly determined by the assumptions imposed on the interaction. For $p > d$, we define

$$\Omega_p := \left\{ \omega \in \Omega \;\middle|\; \|\omega\|_p := \left[\sum_\ell (1+|\ell|)^{-p} |\omega_\ell|^R_{L^R_\beta} \right]^{1/R} < \infty \right\}, \tag{3.4}$$

which is a locally convex Polish spaces with the topology induced by the system of seminorms $||\omega||_p$ and $|\omega_\ell|_{C_\beta}$, $\ell \in \mathbb{L}$. Recall that the parameter $R \geq 2$ describes the (largest possible) order of polynomial growth allowed for $W_{\ell\ell'}$ by Assumption (W). Thereafter we define the subset of *tempered configurations*

$$\Omega^{\mathrm{t}} := \bigcup_{p>0} \Omega_p = \{\omega \in \Omega \mid \exists p = p(\omega) > 0 : \; \|\omega\|_p < \infty \; \} \tag{3.5}$$

and, respectively, the subset of *tempered measures*

$$\mathcal{P}^{\mathrm{t}}(\Omega) := \{\mu \in \mathcal{P}(\Omega) \mid \exists p = p(\mu) > 0 : \; \mu(\Omega_p) = 1\}. \tag{3.6}$$

4. Euclidean Gibbs measures

First, we introduce a free Gaussian measure χ on the spin space C_β, which in the Euclidean representation corresponds to a single ν-dimensional quantum harmonic oscillator of mass $m > 0$ and rigidity $a > 0$. With this aim, in the Hilbert space L^2_β we consider the Laplace–Beltrami operator $A := \left(-m\frac{\mathrm{d}^2}{\mathrm{d}\tau^2} + a\mathbf{1}\right) \otimes \mathrm{Id}_{\mathbb{R}^\nu}$, where $\mathrm{Id}_{\mathbb{R}^\nu}$ denotes the identity matrix in $\mathbb{R}^\nu$. Since its inverse A^{-1} is of trace class, the Fourier transform

$$\int_{L^2_\beta} \exp i(\varphi, \upsilon)_{L^2_\beta} \mathrm{d}\chi(\upsilon) = \exp\left\{ -\frac{1}{2}(A^{-1}\varphi, \varphi)_{L^2_\beta} \right\}, \quad \varphi \in L^2_\beta, \tag{4.1}$$

uniquely defines a Gaussian measure χ on $(L^2_\beta, \mathcal{B}(L^2_\beta))$. As well known(cf. [28]), χ is supported by the spaces of Hölder continuous loops $C^\sigma_\beta \subset C_\beta$ with $\sigma \in (0, 1/2)$, and there exists $\lambda_\sigma > 0$ such that

$$\int_{L^2_\beta} \exp\left(\lambda_\sigma |\upsilon|^2_{C^\sigma_\beta}\right) \chi(\mathrm{d}\upsilon) < \infty. \tag{4.2}$$

Heuristically, the Euclidean Gibbs measures μ we are interested in have the following representation

$$\mu(\mathrm{d}\omega) := Z^{-1} \exp\{-I(\omega)\} \prod_\ell \chi(\mathrm{d}\omega_\ell), \tag{4.3}$$

where the map

$$\Omega \ni \omega \longmapsto I(\omega) := \int_{S_\beta} \left[\sum_\ell V_\ell(\omega_\ell) + \sum_{\ell,\ell'} W_{\ell\ell'}(\omega_\ell, \omega_{\ell'}) \right] d\tau$$

might be viewed as a potential energy functional describing an interacting system of loops $\omega_\ell \in C_\beta$ indexed by $\ell \in \mathbb{L}$. Following the standard Dobrushin–Lanford–Ruelle (*DLR*) route (cf., e.g.,[16]), a rigorous meaning to μ can be given through their local specification $\{\pi_\Lambda\}_{\Lambda \Subset \mathbb{L}}$. In the present context, this is a family of measure kernels

$$\mathcal{B}(\Omega) \times \Omega \ni (B, \xi) \mapsto \pi_\Lambda(B|\xi) \in [0,1],$$

$$\pi_\Lambda(B|\xi) := Z_\Lambda^{-1}(\xi)_{\Omega_\Lambda} \exp\{-I_\Lambda(\omega_\Lambda|\xi)\} \mathbf{1}_{B \bigcap \Omega^{\mathrm{t}}}(\omega_\Lambda \times \xi_{\Lambda^c}) \prod_{\ell \in \Lambda} \chi(\mathrm{d}\omega_\ell), \tag{4.4}$$

(where $\mathbf{1}_B$ denotes the indicator on B). Here

$$Z_\Lambda(\xi) := \int_{\Omega_\Lambda} \exp\left[-I_\Lambda(\omega_\Lambda|\xi)\right] \chi_\Lambda(\mathrm{d}\omega_\Lambda), \tag{4.5}$$

is the normalization factor and

$$\begin{aligned} I_\Lambda(\omega_\Lambda|\xi) := \sum_{\ell \in \Lambda} \int_{S_\beta} V_\ell(\omega_\ell(\tau)) d\tau + \frac{1}{2} \sum_{\ell, \ell' \in \Lambda} \int_{S_\beta} W_{\ell,\ell'}(\omega_\ell(\tau), \omega_{\ell'}(\tau)) d\tau \\ + \sum_{\ell \in \Lambda,\ \ell' \in \Lambda^c} \int_{S_\beta} W_{\ell,\ell'}(\omega_\ell(\tau), \xi_{\ell'}(\tau)) d\tau \end{aligned} \tag{4.6}$$

is the interaction in the volume Λ under the boundary condition $\xi_{\Lambda^c} := (\xi_{\ell'})_{\ell' \in \Lambda^c}$. Obviously, (4.4)–(4.6) make sense for the potentials dealt with. By the construction, the family $\{\pi_\Lambda\}_{\Lambda \Subset \mathbb{L}}$ is consistent, that is for all $B \in \mathcal{B}(\Omega)$ and $\xi \in \Omega$

$$\int_\Omega \pi_\Lambda(B|\omega) \pi_{\Lambda'}(\mathrm{d}\omega|\xi) = \pi_{\Lambda'}(B|\xi), \quad \Lambda \subseteq \Lambda', \tag{4.7}$$

Moreover, by (4.2) it follows that for any $\sigma \in (0, 1/2)$ and $\kappa > 0$

$$\int_\Omega \exp\left\{ \sum_{\ell \in \Lambda} \left(\lambda_\sigma |\omega_\ell|^2_{C^\sigma_\beta} + \kappa |\omega_\ell|^R_{L^R_\beta} \right) \right\} \pi_\Lambda(\mathrm{d}\omega|\xi) < \infty, \tag{4.8}$$

where λ_σ is the same as in (4.2).

Definition 4.1. *A probability measure $\mu \in \mathcal{P}(\Omega)$ is called Euclidean Gibbs measure (corresponding to the quantum system (2.1) at inverse temperature $\beta > 0$) if it satisfies the DLR equilibrium equation*

$$\int_\Omega \pi_\Lambda(B|\omega) \mu(\mathrm{d}\omega) = \mu(B), \quad \textit{for all } \Lambda \Subset \mathbb{L} \textit{ and } B \in \mathcal{B}(\Omega). \tag{4.9}$$

Fixing $\beta > 0$, let $\mathcal{G}$ denote the set of all such measures μ. By the above definition $\mu(\Omega^{\mathrm{t}}) = 1$ for each $\mu \in \mathcal{G}$. We shall be concerned with the subset of *tempered Gibbs measures*

$$\mathcal{G}^{\mathrm{t}} := \mathcal{G} \cap \mathcal{P}^{\mathrm{t}}(\Omega) \ = \ \{\mu \in \mathcal{G} \mid \exists p = p(\mu) > d : \ \mu(\Omega_p) = 1\}. \tag{4.10}$$

For $p > d$, on the set of probability measures $\mathcal{P}(\Omega_p)$ we introduce the weak topology $\mathcal{W}_p$, standardly defined by means of bounded continuous functions $f \in C_{\mathrm{b}}(\Omega_p)$. An important observation is that, under our hypotheses on the interaction, each $\mathcal{W}_p$-accumulation point $\mu \in \mathcal{P}(\Omega_p)$ of the family $\{\pi_\Lambda(\cdot|\xi) \mid \Lambda \Subset \mathbb{L},\ \xi \in \Omega_p\}$, as $\Lambda \nearrow \mathbb{L}$, is the tempered Euclidean Gibbs measure.

5. Formulation of the Main Results

The theorems below provide us with basic information for any further investigation of the Euclidean Gibbs measures. We suppose that Assumptions (W), (J), and (V) are fulfilled without mentioning this again in the formulations of our statements.

Theorem 5.1. (Existence) *For all values of $\beta > 0$, the set of tempered Euclidean Gibbs measures $\mathcal{G}^{\mathrm{t}}$ is nonempty.*

The next theorem says that the tempered Euclidean Gibbs measures satisfy an exponential moment estimate in the state space C_β^σ, similar to the one (4.2) valid for the free loop measure γ.

Theorem 5.2. (À-priori estimate) *For every $\sigma \in (0, 1/2)$ and $\kappa > 0$, there exists $C := C_\sigma(\beta, \kappa) > 0$, such that uniformly for any $\ell \in \mathbb{L}$ and $\mu \in \mathcal{G}^{\mathrm{t}}$*

$$\int_\Omega \exp\left(\lambda_\sigma |\omega_\ell|^2_{C_\beta^\sigma} + \kappa |\omega_\ell|^R_{L_\beta^R}\right) \mu(\mathrm{d}\omega) \le C. \tag{5.1}$$

This bound is called à-priori, since it holds independently of the existence result, whereby the constant $C_\sigma(\kappa)$ can be calculated explicitly in terms of parameters of the interaction. The estimate (5.1) plays a crucial role in the theory of the set $\mathcal{G}^{\mathrm{t}}$ and gives more information about the regularity and support properties of its elements.

Corollary 5.3. (Compactness) *$\mathcal{G}^{\mathrm{t}}$ is the $\mathcal{W}_p$-compact subset in any Ω_p, $p > d$..*

Corollary 5.4. (Regularity of finite-volume projections) *For each $\mu \in \mathcal{G}^{\mathrm{t}}$, its projections $\mu_\Lambda := \mu \circ \mathbb{P}_\Lambda^{-1}$ under the mappings $\mathbb{P}_\Lambda : \omega \mapsto \omega_\Lambda$, $\Lambda \Subset \mathbb{L}$, are absolutely continuous with respect to the Gaussian measures $\chi_\Lambda(\mathrm{d}\omega_\Lambda) := \prod_{\ell\in\Lambda} \chi(\mathrm{d}\omega_\ell)$ on $(\Omega_\Lambda, \mathcal{B}(\Omega_\Lambda))$. The corresponding Radon–Nikodym derivatives obey the Ruelle-type bound*

$$\frac{\mathrm{d}\mu_\Lambda}{\mathrm{d}\chi_\Lambda}(\omega_\Lambda) \le \exp\left(-\mathcal{K}\sum\nolimits_{\ell\in\Lambda} |\omega_\ell|^R_{L_\beta^R} + \mathcal{L}_\Lambda\right) \tag{5.2}$$

with an arbitrary $\mathcal{K} \in (0, \infty)$ and a certain $\mathcal{L}_\Lambda := \mathcal{L}_\Lambda(\beta, K) \in \mathbb{R}$, which can be chosen the same for all μ and ℓ.

Corollary 5.5. (Lebowitz–Presutti Support) *For every $\sigma \in (0, 1/2)$, there exists $b := b_\sigma(\beta) > 0$ such that all $\mu \in \mathcal{G}^{\mathrm{t}}$ are supported by the Borel subset*

$$\begin{aligned}\Xi_\sigma(b) := \Big\{\omega \in \Omega \mid (\forall \ell_0 \in \mathbb{L})\ (\exists \Lambda_{\omega,\ell_0} \Subset \mathbb{L})\ (\forall \ell \in \Lambda^c_{\omega,\ell_0}) : \\ |\omega_\ell|^2_{C_\beta^\sigma} \le b \log(1 + |\ell - \ell_0|)\Big\}.\end{aligned} \tag{5.3}$$

Finally, we present our uniqueness result, which says that the set of tempered Gibbs measures consists of exactly one point, provided the strength of the interaction is small. Although such result is rather expected (e.g., via cluster expansions), so far its direct analytical proof was not known for superquadratic interactions.

Theorem 5.6. (Uniqueness) *Consider the spin system (2.2) on the lattice $\mathbb{L} := \mathbb{Z}^d$. Let the matrix $(J_{\ell\ell'})_{\mathbb{L}\times\mathbb{L}}$ in Assumption (J) be translation invariant and have finite range (i.e., $J_{\ell\ell'} := J_{\ell'-\ell} \ge 0$ for all $\ell, \ell \in' \mathbb{L}$, and there exists $r \ge 1$ such that $J_{\ell\ell'} := 0$ if $|\ell - \ell'| > r$). Then, for any $\beta > 0$ one finds a proper $\mathcal{J}(\beta) > 0$, such that for all values of $||J||_0 \le \mathcal{J}(\beta)$ the corresponding set $\mathcal{G}^{\mathrm{t}}$ is singleton.*

Comments: (i) Even the initial question of whether there exists any $\mu \in \mathcal{G}^{\mathrm{t}}$, to which we give a positive answer by Theorem 5.1, is not evident at all. A standard tool for proving existence is the fundamental Dobrushin criterion (cf. Theorem 1 in [13]). Contrary to the known results in the classical case (cf. [9, 12, 27]), the validity of Dobrushin's criterion for quantum systems with the (infinite-dimensional) spin spaces L^R_β, C^σ_β was not covered by any previous work. Furthermore, all proofs in the quoted papers were designed for scalar ferromagnetic models, so that their extension to multi- (or infinite-) dimensional spins and general interactions seems to be impossible. In Lemma 6.1 below we shall present *a simple new argument*, which applies universally both to the classical and quantum cases. Furthermore, it straightforwardly extends to general many-particle interactions (e.g., not necessarily translation invariant and possibly having unbounded order and infinite range) that are beyond reach by other methods (cf. Subsections 7.2).

(ii) Theorem 5.2 contributes to another important problem of getting *uniform estimates* on Gibbs measures in terms of parameters of the interaction. This problem was initially posed for classical lattice systems in [9]. For the quantum anharmonic crystals, some weaker estimates were obtained in the so-called stochastic and analytical approaches, alternative to the traditional one based on the DLR equations (see[3], [4], [7]).

(iii) The uniqueness problem for $\mu \in \mathcal{G}^{\mathrm{t}}$ could be treated in the particular case of $R = 2$ by means of the another renown criterion of Dobrushin, see Theorem 4 in [13] and its applications to the quantum lattice systems in [5], [6]. However, this criterion is typically not applicable to the pair interactions $W_{\ell\ell'}$ growing fastly than quadratic as considered here. For this reason we use a proper modification of the Dobrushin uniqueness criterion to lattice systems with non-compact spin spaces, which was suggested by Dobrushin and Pechersky in [14] (for more details see the next section).

6. Proof of Theorems

Here we briefly sketch the proofs of our main theorems and outline the basic ideas. The key technical result is the following lemma, which gives an exponential bound for probability kernels $\pi_\ell(\mathrm{d}\omega|\xi)$ subject to the fixed boundary condition $\xi \in \Omega^{\mathrm{t}}$. To shorten notation, we write π_ℓ instead of $\pi_{\{\ell\}}$.

Lemma 6.1. *For every $\sigma \in (0, 1/2)$ and $\kappa > 0$, there exists a corresponding $\Upsilon = \Upsilon_\sigma(\beta, \kappa) > 0$ such that for all $\ell \in \mathbb{L}$ and $\xi \in \Omega^{\mathrm{t}}$,*

$$\int_\Omega \exp\left\{\lambda_\sigma |\omega_\ell|^2_{C^\sigma_\beta} + \kappa |\omega_\ell|^R_{L^R_\beta}\right\} \pi_\ell(\mathrm{d}\omega|\xi) \le \exp\left\{\Upsilon + \sum_{\ell'} J_{\ell\ell'} |\xi_{\ell'}|^R_{L^R_\beta}\right\}. \tag{6.1}$$

Here $\lambda_\sigma > 0$ is the same as in (4.8).

By Jensen's inequality, one readily gets from (6.1) the following

Corollary 6.2. (Dobrushin's bound) *For all $\ell \in \mathbb{L}$ and $\xi \in \Omega^{\mathrm{t}}$*

$$\int_\Omega h(\omega_\ell)\pi_\ell(\mathrm{d}\omega|\xi) \le C + \sum_{\ell'} I_{\ell\ell'} |\xi_{\ell'}|^R_{L^R_\beta} \le C + \sum_{\ell'} I_{\ell\ell'} h(\xi_{\ell'}), \tag{6.2}$$

with $C := \Upsilon/\kappa$, $I_{\ell\ell'} := J_{\ell\ell'}/\kappa$, and the compact function $h : C_\beta \to \mathbb{R}$,

$$h(\omega_\ell) := \lambda_\sigma \kappa^{-1} |\omega_\ell|^2_{C^\sigma_\beta} + |\omega_\ell|^R_{L^2_\beta}. \tag{6.3}$$

Note that the function $h(\omega_\ell)$ is a sum of two nonlinear terms, the first of which, with $|\omega_\ell|^2_{C^\sigma_\beta}$, guarantees the compactness on C_β, whereas the second one, with $|\omega_\ell|^R_{L^R_\beta}$, controls the growth of the pair interaction $W_{\ell,\ell'}$. It seem to the first explicit example of a compact function satisfying Dobrushin's criterion on loop spaces.

Proof of Lemma 6.1 By Assumptions (W) and (J) one has for all ω, $\xi \in \Omega^{\mathrm{t}}$

$$\sum_{\ell'(\neq\ell)} \int_{S_\beta} |W_{\ell,\ell'}(\omega_\ell(\tau),\xi_{\ell'}(\tau))|\mathrm{d}\tau \leq \frac{||\mathbf{J}||_0}{2}\left(\beta + |\omega_\ell|^R_{L^R_\beta}\right) + \frac{1}{2}\sum_{\ell'} J_{\ell\ell'}|\xi_{\ell'}|^R_{L^R_\beta}. \tag{6.4}$$

By this estimate and the definition (4.4) of $\pi_\ell(\mathrm{d}\omega|\xi)$

$$\begin{aligned}&\int_\Omega \exp\left\{\lambda_\sigma|\omega_\ell|^2_{C^\sigma_\beta} + \kappa|\omega_\ell|^R_{L^R_\beta}\right\}\pi_\ell(\mathrm{d}\omega|\xi) \\ &\quad\leq \frac{X_\ell}{Y_\ell}\exp\left\{\sum_{\ell'(\neq\ell)} J_{\ell\ell'}|\xi_{\ell'}|^R_{L^R_\beta} + ||\mathbf{J}||_0 C_W\beta\right\},\end{aligned} \tag{6.5}$$

where

$$X_\ell := \int_\Omega \exp\left\{\lambda_\sigma|\omega_\ell|^2_{C^\sigma_\beta} + (\kappa + ||\mathbf{J}||_0/2)\,|\omega_\ell|^R_{L^R_\beta} - \int_{S_\beta} V_\ell(\omega_\ell(\tau))\mathrm{d}\tau\right\}\chi(\mathrm{d}\omega_\ell), \tag{6.6}$$

$$Y_\ell := \int_\Omega \exp\left\{-\frac{||\mathbf{J}||_0}{2}|\omega_\ell|^R_{L^R_\beta} - \int_{S_\beta} V_\ell(\omega_\ell(\tau))\mathrm{d}\tau\right\}\chi(\mathrm{d}\omega_\ell). \tag{6.7}$$

Now we use the upper and lower bounds in (2.3) to show that respectively $\sup_\ell X_\ell < \infty$ and $\inf_\ell Y_\ell > 0$. Finally, taking into account (4.2), we arrive at (6.1). $\square$

The next step is to get similar moment estimates for $\pi_\Lambda(\mathrm{d}\omega|\xi)$ uniformly in volumes $\Lambda \Subset \mathbb{L}$. We set (cf. (6.1))

$$n_\ell(\Lambda|\xi) := \log\left\{\int_\Omega \exp\left(\lambda_\sigma|\omega_\ell|^2_{C^\sigma_\beta} + \kappa|\omega_\ell|^R_{L^R_\beta}\right)\pi_\Lambda(\mathrm{d}\omega|\xi)\right\} \geq 0, \tag{6.8}$$

which makes sense by (4.8).

Lemma 6.3. *Given $\sigma \in (0, 1/2)$ and $p > d$, there exists a finite $\Psi := \Psi_{\sigma,p}(\beta,\kappa) > 0$ such that*

$$\limsup_{\Lambda\nearrow\mathbb{L}}\left[\sum_{\ell\in\Lambda} n_\ell(\Lambda|\xi)\cdot(1+|\ell_0-\ell|)^{-p}\right] \leq \Psi, \tag{6.9}$$

uniformly for all $\ell_0 \in \mathbb{L}$ and $\xi \in \Omega_p$. This implies, in particular, that for all $\ell \in \mathbb{L}$ and $\xi \in \Omega_p$

$$\limsup_{\Lambda\nearrow\mathbb{L}}\int_\Omega \exp\left(\lambda_\sigma|\omega_\ell|^2_{C^\sigma_\beta} + \kappa|\omega_\ell|^R_{L^R_\beta}\right)\pi_\Lambda(\mathrm{d}\omega|\xi) = \exp\Psi. \tag{6.10}$$

Proof. Integrating both sides of (6.1) with respect to the measure $\pi_\Lambda(\mathrm{d}\omega|\xi)$ and taking into account (4.7), we arrive at

$$\begin{aligned} n_\ell(\Lambda|\xi) &\leq \Upsilon + \sum_{\ell'\in\Lambda^c}|J_{\ell\ell'}|\cdot|\xi_{\ell'}|^R_{L^R_\beta} + \log\left\{\int_\Omega\exp\left(\sum_{\ell'\in\Lambda}J_{\ell\ell'}|\omega_{\ell'}|^R_{L^R_\beta}\right)\pi_\Lambda(\mathrm{d}\omega|\xi)\right\} \\ &\leq \Upsilon + \sum_{\ell'\in\Lambda^c}J_{\ell\ell'}|\xi_{\ell'}|^R_{L^R_\beta} + \kappa^{-1}\sum_{\ell'\in\Lambda}J_{\ell\ell'}n_{\ell'}(\Lambda|\xi). \qquad (6.11)\end{aligned}$$

Note that in the last line we have used a multiple Hölder's inequality, which was possible due to the choice of $\kappa > ||J||_p$. After summing over $\ell \in \Lambda$, the estimate (6.11) leads to the required one

$$\limsup_{\Lambda \nearrow \mathbb{L}} n_{\ell_0}(\Lambda|\xi) \leq \limsup_{\Lambda \nearrow \mathbb{L}} \left[\sum_{\ell \in \Lambda} n_\ell(\Lambda|\xi) \cdot (1 + |\ell_0 - \ell|)^{-p} \right] \leq \frac{\varUpsilon}{1 - \kappa^{-1}||\mathbf{J}||_p} \cdot \sup_{\ell_0} \sum_{\ell} (1 + |\ell_0 - \ell|)^{-p} =: \varPsi. \tag{6.12}$$

□

Now we demonstrate elementary proofs of Theorems 5.1 and 5.2 based on the key estimate (6.1)

Proof of Theorem 5.1: Let us introduce the scale of Banach spaces, indexed by $\sigma \in (0, 1/2)$ and $p > d$,

$$\varOmega_{\sigma,p} := \left\{ \omega \in \varOmega \;\middle|\; ||\omega||_{\sigma,p} := \left[\sum_{\ell} (1 + |\ell|)^{-p} |\omega_\ell|^2_{C^\sigma_\beta} \right]^{1/2} < \infty \right\}. \tag{6.13}$$

Note that the embeddings $\varOmega_{\sigma,p} \hookrightarrow \varOmega_{p'}$ are compact if $p' > pR/2$. By (6.9) we have that

$$\sup_{\Lambda \Subset \mathbb{L}} \int_{\varOmega} ||\omega||^2_{\sigma,p} \pi_\Lambda(\mathrm{d}\omega|0) < \infty, \tag{6.14}$$

which by Prokhorov's criterion implies that the family $\{\pi_\Lambda(\mathrm{d}\omega|0)\}_{\Lambda \Subset \mathbb{L}}$ is $\mathcal{W}_{p'}$-relatively compact. All its accumulation points surely belong to $\mathcal{G}^{\mathrm{t}}$. □

Proof of Theorem 5.2: Let $p > d$ be fixed, and consider any $\mu \in \mathcal{G}^{\mathrm{t}}$ supported by $\varOmega_p$. Then, by means of (4.9), (6.10), and Fatou's lemma, one has

$$\begin{aligned} &\int_{\varOmega} \exp\left(\lambda_\sigma |\omega_\ell|^2_{C^\sigma_\beta} + \kappa |\omega_\ell|^R_{L^R_\beta} \right) \mu(\mathrm{d}\omega) \\ &= \lim_{N \to \infty} \int_{\varOmega} \exp\left[\min\left\{ \lambda_\sigma |\omega_\ell|^2_{C^\sigma_\beta} + \kappa |\omega_\ell|^R_{L^R_\beta};\; N \right\} \right] \mu(\mathrm{d}\omega) \\ &\leq \int_{\varOmega} \left[\limsup_{\Lambda \nearrow \mathbb{L}} \int_{\varOmega} \exp\left\{ \min \lambda_\sigma |\omega_\ell|^2_{C^\sigma_\beta} + \kappa |\omega_\ell|^R_{L^R_\beta} \right\} \pi_\Lambda(\mathrm{d}\omega|\xi) \right] \mu(\mathrm{d}\xi) \leq \exp \varPsi_{\sigma,p}(\beta, \kappa). \end{aligned} \tag{6.15}$$

This implies by Chebyshev's inequality that any $\mu \in \mathcal{G}^{\mathrm{t}}$ is actually supported by $\bigcap_{p'>d} \varOmega_{p'}$. Hence, (6.15) yields the desired estimate (5.1) with the constant $C_\sigma(\beta, \kappa) = \exp \varPsi_{\sigma,p}(\beta, \kappa)$, which is the same for all $\mu \in \mathcal{G}^{\mathrm{t}}$. □

The proofs of Corollaries 5.3–5.5 are rather standard and develop the corresponding ideas from [9], [12], and [24].

The **proof of Theorem 5.6** is based on the uniqueness criterion for lattice Gibbs fields with non-compact spins, which was suggested by Dobrushin and Pechersky (cf. Theorem 1 in [14]). It requires following two conditions to be fulfilled for the probability kernels $\pi_\ell(\mathrm{d}x|y)$. The first one is a stronger version of Dobrushin's compactness condition (6.2), which should hold with a contractive matrix $(I_{\ell-\ell'})_{\mathbb{L}\times\mathbb{L}}$ whose norm $||\mathbf{I}||_0$ is smaller than a certain constant $\gamma(d, r) < 1$ (depending on the geometry of the lattice and radius of the interaction). The second condition is on the variation probability distance in C_β between the one-point conditional distributions

$\mu_\ell(\mathrm{d}x_\ell|y) := \pi(\mathrm{d}x_\ell|y) \circ \mathbb{P}_\ell^{-1}$. For given $\mathcal{R} \geq 0$ one has to check that there exists a strictly contractive matrix $(K_{\ell-\ell'})_{\mathbb{L}\times\mathbb{L}}$, such that

$$\mathrm{D}_{\mathrm{var}}\left(\mu_\ell(\mathrm{d}\omega_\ell|\xi), \mu_\ell(\mathrm{d}\omega_\ell|\tilde{\xi})\right) \leq K_{\ell-\ell'} \tag{6.16}$$

for each pair of points $\ell \neq \ell'$ and all configurations $\xi, \tilde{\xi} \in \Omega$ differing only at ℓ' and satisfying $\sup_{\ell\in\mathbb{L}} \{h(y_\ell),\ h(\tilde{y}_\ell)\} \leq \mathcal{R}$. The validity of the both condition is obvious for small $||\mathbf{J}||_0$, once we have proved Lemma 6.1.

7. Possible generalizations and concluding remarks

7.1. **The case of P=R.** Suppose that Assumptions (W), (J), and (V) hold with $P = R \geq 2$. Then, for fixed $\beta > 0$, the statements of Theorems 5.1 and 5.2 are still true if $||\mathbf{J}||_0 < \beta A_V$, where $A_V > 0$ is the (largest possible) constant in (2.5). In the corresponding moment estimates (5.1) and (6.1) one may take any $0 < \kappa < \beta A_V - ||\mathbf{J}||_0$. To this end, on the spaces Ω_p and $\Omega_{\sigma,p}$ we introduce new systems of norms, indexed by $\delta > 0$,

$$\|\omega\|_{p,\delta} := \left[\sum_\ell (1+\delta|\ell|)^{-p}|\omega_\ell|^R_{L^R_\beta}\right]^{1/R}, \tag{7.1}$$

$$\|\omega\|_{p,\sigma,\delta} := \left[\sum_\ell (1+\delta|\ell|)^{-p}|\omega_\ell|^2_{C^R_\beta}\right]^{1/2},$$

which are equivalent to the initial ones $\|\omega\|_p$ and $\|\omega\|_{p,\sigma}$ (cf. (3.4) and. 6.13). Given $p > d$, we next choose small enough $\delta = \delta(p) > 0$ so that

$$||\mathbf{J}||_{p,\delta} := \sup_\ell \sum_{\ell'} (1+\delta|\ell'|)^p J_{\ell\ell'} < A_V. \tag{7.2}$$

Thereafter, we go through the above proofs and use everywhere the system of weights $(1+\delta|\ell|)^{-p}$, the norms (7.1), and the relation (7.2).

7.2. **Many-particle interactions.** Our results extend to quantum systems with many-particle interactions of possibly infinite range and infinite order**.** Such systems are described by the heuristic Hamiltonian

$$H = -\frac{1}{2m}\sum_\ell |p_l|^2 + \frac{a}{2}\sum_\ell |q_\ell|^2 + \sum_\ell V_\ell(q_\ell) + \sum_{n=2}^N \frac{1}{n!}\sum_{\ell_1,\dots,\ell_n} W_{\ell_1\dots\ell_n}(q_{\ell_l},\dots,q_{\ell_n}), \tag{7.3}$$

where the n-particle interaction potentials (taken over all ordered sets consisting of mutually different points $\ell_1,\dots,\ell_n$ with $2 \leq n \leq N \leq +\infty$) are given by continuous symmetric functions $W_{\ell_1\dots\ell_n} : \mathbb{R}^{dn} \to \mathbb{R}^\nu$. Then Theorems 5.1 and 5.2 are true under Assumptions (V), (W^*), and (J^*), where (W^*) and (J^*) respectively are the following modification of (W) and (J):

(W*): There exist $R \geq 2$ and symmetric matrices $\{J_{\ell_1\dots\ell_n} \geq 0\}_{\mathbb{L}^n}$, such that

$$|W_{\ell_1\dots\ell_n}(q_1,\dots,q_n)| \leq \frac{1}{2}J_{\ell_1\dots\ell_n}\left(1+\sum_{m=1}^n |q_m|^R\right) \quad \text{for all } q_1,\dots,q_n \in \mathbb{R}^\nu. \tag{7.4}$$

(J*): The matrices $\{J_{\ell_1 \dots \ell_n}\}_{\mathbb{L}^n}$, $n = 2, ..., N$, are fastly decreasing, that is for any $p \geq 0$

$$||\mathbf{J}|| := \sum_{n=2}^{N} n^2 \sup_{\ell_1} \left\{ \sum_{\ell_1, \dots, \ell_n} J_{\ell_1 \dots \ell_n} \left(1 + \sum\nolimits_{m=1}^{n} |\ell_1 - \ell_m|\right)^p \right\} < \infty. \tag{7.5}$$

The proofs are similar to those carried before.

References

[1] S. Albeverio and R. Høegh-Krohn: Homogeneous random fields and quantum statistical mechanics, *J. Funct. Anal.* 19, 241–272 (1975)..

[2] S. Albeverio, Yu. G. Kondratiev, Yu. Kozitsky, M. Röckner: Euclidean Gibbs states of quantum lattice systems, *Rev. Math. Phys.* 14, 1–67 (2002).

[3] S. Albeverio, Yu. G. Kondratiev, T. Pasurek, M. Röckner: Euclidean Gibbs measures on loop spaces: existence and a priori estiamtes, *Ann. Probab.* 32, 153–190 (2004).

[4] S. Albeverio, Yu. G. Kondratiev, T. Pasurek, M. Röckner: A priori estimates and existence for Euclidean Gibbs states, to appear in *Trans. Moscow Math. Society* 67 (2006).

[5] S. Albeverio, Yu. Kondratiev, M. Röckner, T. Tsikalenko: Uniqueness of Gibbs states for quantum lattice systems. *Prob. Theory Rel. Fields* 108, 193–218 (1997).

[6] S. Albeverio, Yu. Kondratiev, M. Röckner, T. Tsikalenko: Dobrushin's uniqueness for quantum lattice systems with nonlocal interaction, *Comm. Math. Phys.* 189, 621–630 (1997).

[7] S. Albeverio, Y. G. Kondratiev, M. Röckner, T. Tsikalenko: Glauber dynamics for quantum lattice systems, *Rev. Math. Phys.* 13, 51–124 (2001).

[8] L. Bierke, J. Fröhlich: KMS,etc., *Rev. Math. Phys.* 14, 829–871 (2002).

[9] J. Bellissard, R. Høegh-Krohn: Compactness and the maximal Gibbs states for random Gibbs fields on a lattice, *Commun. Math. Phys.* 84, 297–327 (1982).

[10] V. S. Barbulyak, Yu. G. Kondratiev: Functional integrals and quantum lattice systems, *Reports Nat. Acad. Sci. of Ukraine*, No. 8, 31–34; No. 9, 38–40; No. 10, 19–21 (1991).

[11] O. Bratteli, D. W. Robinson: *Operator Algebras and Quantum Statistical Mechanics, I, II*, Springer, Berlin Heidelberg New York (1981).

[12] M. Cassandro, E. Olivieri, A. Pellegrinotti, E. Presutti: Existence and uniqueness of DLR measures for unbounded spin systems, *Z. Wahrsch. verw. Gebiete* 41, 313–334 (1978).

[13] R. L. Dobrushin: Prescribing a system of random variables by conditional distributions, *Theory Prob. Appl.* 15, 458–486 (1970).

[14] R. L. Dobrushin, E. A. Pechersky: A criterion of the uniqueness of Gibbs field in the non-compact case, *Lecture Notes Math.* 1021, 97–110 (1982).

[15] W. Driesler, L. Landau, J. F. Perez: Estimates of critical length and critical temperatures for classical and quantum lattice systems, *J. Stat. Phys.* 20**,** 123–162 (1979).

[16] H.-O. Georgii: *Gibbs Measures and Phase Transitions*, Studies in Mathematics 9, Walter de Gruyter, Berlin New York, 1988.

[17] J. Glimm, A. Jaffe: *Quantum physics. A functional integral point of view*, Springer, Berlin – Heidelberg – New York, 1981.

[18] R. Gielerak, L. Jakobczyk, and R. Olkiewicz: Reconstuction of Kubo–Martin–Schwinger structure from Euclidean Green functions, *J. Math. Phys.* 35, 3726–3744 (1994).

[19] A. Klein, L. Landau: Stochastic processes associated with KMS states, *J. Funct. Anal.* 42, 368–428 (1981).

[20] Yu. Kozitsky, T. Pasurek: Gibbs states of interacting systems of quantum anharmonic oscillators; to appear in *Letters Math. Phys.*

[21] Yu. Kozitsky, T. Pasurek: Euclidean Gibbs measures of interacting quantum anharmonic oscillators, Preprint arXiv.org, math-ph/0609045, 60 pages (2006).

[22] J. L. Lebowitz, E. Presutti: Statistical mechanics of systems of unbounded spins, *Commun. Math. Phys.* 50, 195–218 (1976).

[23] R. A. Minlos, A. Verbeure, V. Zagrebnov: A quantum crystal model in the light mass limit: Gibbs state, *Rev. Math. Phys.* 12, 981–1032 (2000).

[24] T. Pasurek: Spin systems with superquadratic interactions: existence and high/low temperature uniqueness, Preprint (2006), 50 pages.

[25] Y. M. Park, H. J. Yoo: A characterization of Gibbs states of lattice boson systems, *J. Stat. Phys.* 75, 215–239 (1994).
[26] D. Ruelle: *Statistical Mechanics. Rigorous Results*, Benjamin, New York Amsterdam, 1969.
[27] Ya. Sinai: *Theory of Phase Transitions. Rigorous Results*, Pergamon Press, Oxford (1982).
[28] B. Simon: *Functional Integration and Quantum Physics*, Academic Press, New York London, 1979.

Fakultät für Mathematik, Universität Bielefeld, D-33615 Bielefeld, Germany
E-mail address: tpasurek@math.uni-bielefeld.de