Dr. Neil Manibo

Fakultät für Mathematik,
Universität Bielefeld

Sommersemester 2026
Bielefeld, 20.05.2026

Lineare Operatoren in Hilberträumen

Exercise Sheet 6

(18) Two Hilbert spaces X and Y are said to be unitarily isomorphic if there exists a unitary map $U\colon X \to Y$. Show that X and Y are unitarily isomorphic if and only if

$$\dim X = \dim Y.$$

(2 Points)

(19) Consider the operator $P = \frac{1}{i}\frac{\mathrm{d}}{\mathrm{d}x}$. Let

$$H^1(J) := \{f \in L^2(J) : \ f \text{ is absolutely continuous, } f' \in L^2(J)\}.$$

Determine P^* and $D(P^*)$, and compute the deficiency indices $(\gamma_+(P), \gamma_-(P))$ for each of the cases below.

(a) $J = \mathbb{R}$, $D(P) = H^1(J)$

(b) $J = (a, b)$, with $-\infty < a < b < \infty$, $D(P) = \{f \in H^1(J) : \ f(a) = f(b) = 0\}$

(c) $J = (0, \infty)$, $D(P) = \{f \in H^1(J) : \ f(0) = 0\}$

(2+2+2 Points)

(20) A symmetric operator S on a Hilbert space X is called *maximally symmetric* if it admits no proper symmetric extension.

(a) Prove that every maximally symmetric operator is closed.

(b) Prove that a closed symmetric operator is maximally symmetric if and only if at least one of the deficiency indices $(\gamma_+(S), \gamma_-(S))$ is equal to zero.

(c) Which of the operator/s in **(19)** is/are maximally symmetric?

(1+2+1 Points)

Abgabe bis 12 Uhr am Mittwoch, 27.05.2026