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Lineare Operatoren in Hilberträumen

Exercise Sheet 7

(21) Determine all self-adjoint extensions of the operator $P = \frac{1}{i}\frac{\mathrm{d}}{\mathrm{d}x}$ with

$$D(P) = \{f \in H^1(J) : \ f(a) = f(b) = 0\},$$

where $J = (a, b)$, $-\infty < a < b < \infty$.

(3 Points)

(22) Let X and Y be Hilbert spaces. An operator $K \in B(X, Y)$ is called a *Hilbert–Schmidt* operator, if there exists an orthonormal basis $\{e_\alpha : \ \alpha \in A\}$ such that

$$\sum_{\alpha \in A} \|Ke_\alpha\|^2 < \infty.$$

Show that every Hilbert–Schmidt operator is compact.

(2 Points)

(23) Let (X, μ) be a measurable space. An operator $K \in B(L^2(X, \mu))$ is a Hilbert–Schmidt operator if and only if there exists a kernel $k \in L^2(X \times X, \mu \times \mu)$ such that

$$Kf(x) = \int_X k(x, y) f(y) \,\mathrm{d}\mu(y) \quad \mu\text{-a.e.}$$

for all $f \in L^2(X, \mu)$.

Let Δ be the Laplacian in $\mathbb{R}^m$ and $K \subset \mathbb{R}^m$ be a compact set. Show that $\chi_K(-\Delta - z)^{-1}$ is a Hilbert–Schmidt operator, for all $z \in \mathbb{C} \setminus [0, \infty)$. Here, χ_K is the characteristic function of K.

(4 Points)

Abgabe bis 12 Uhr am Mittwoch, 03.06.2026