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Lineare Operatoren in Hilberträumen

Exercise Sheet 11

(34) (a) Let $V\colon \mathbb{R}^n \to \mathbb{R}$ and suppose $V \in L^p(\mathbb{R}^n) + L^\infty(\mathbb{R}^n)$ where

$$p = 2 \quad \text{if } n \leqslant 3, \quad p > \frac{n}{2} \quad \text{if } n \geqslant 4.$$

Show that $T = -\Delta + V$ is a self-adjoint operator. Here, $L^\infty(\mathbb{R}^n)$ is the space of essentially bounded functions (i.e., bounded except for a set of Lebesgue measure zero). Such a potential is called a Kato–Rellich potential.

Hint: Hölder's inequality

(b) Show that the *Yukawa potential*

$$V(x) = -g^2 \frac{\mathrm{e}^{-\mu|x|}}{|x|} \quad \text{with } \mu \geqslant 0,\ g \in \mathbb{R} \quad (\text{in } \mathbb{R}^3)$$

is a Kato–Rellich potential. (Note that $\mu = 0$ gives the Coulomb potential).

(3+3 Points)

(35) Consider the (homogeneous) 2π-periodic Sturm–Liouville problem (on $[0, 2\pi]$)

$$y' - A(x)y = 0 \quad \text{with} \quad A(x) = \begin{pmatrix} 1 & 1 \\ 0 & \frac{\cos(x)+\sin(x)}{(2-\cos(x)+\sin(x))} \end{pmatrix}$$

where $y = (y_1, y_2)^T$.

(a) Find a fundamental system $X(x)$ with $X(0) = \mathbb{I}$ and determine the transfer matrix C.

(b) Determine the characteristic exponents μ_i and the matrix B. Does the problem admit a 2π-periodic solution?

(c) Determine the Floquet representation $X(x) = Q(x)\mathrm{e}^{Bx}$ of $X(x)$.

Hint: For (a), start with the differential equation for y_2 and solve the solution for y_1 using the solution for y_2.

(4 Points)

Abgabe bis 12 Uhr am Mittwoch, 08.07.2026