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Lineare Operatoren in Hilberträumen

Exercise Sheet 12

(36) Prove parts (c) and (d) of Corollary 5.12 in the lecture notes.

(2 Points)

(37) Consider the potential

$$V(x) = v\left(\frac{x}{|x|}\right)\frac{\exp(-a|x|)}{|x|},$$

where $a \geqslant 0$ und $v\colon S^{d-1} \to \mathbb{R}$ is continuous (note that if v is a constant function, then V is a Yukawa potential). Consider the operator $T = -\Delta + V$.

(a) Show that T has no eigenvalues if $v \geqslant 0$.

(b) Show that T has no eigenvalues in $[-a \min v, \infty)$ if v admits negative values.

Hint: Apply Corollary 5.12.

(2+2 Points)

(38) Show that the sequence $(\mu_n(T))_{n\geqslant 1}$ from the min-max theorem is non-decreasing.

(3 Points)

(39) Let $V \in L^2(\mathbb{R})$ be real-valued and of compact support. Suppose $\int V(x)\mathrm{d}x < 0$. Then the operator

$$T = -\frac{d}{dx^2} + V(x)$$

has at least one negative eigenvalue.

Hint: Consider $\phi \in C_c^\infty(\mathbb{R})$ with $\phi(x) = 1$ for $|x| \leqslant 1$ and $\phi_n(x) := \phi(x/n)$. It suffices to show that $\langle \phi_n, T\phi_n\rangle < 0$ for large enough n (apply the min-max theorem).

(4 Points)

Abgabe bis 12 Uhr am Mittwoch, 15.07.2026