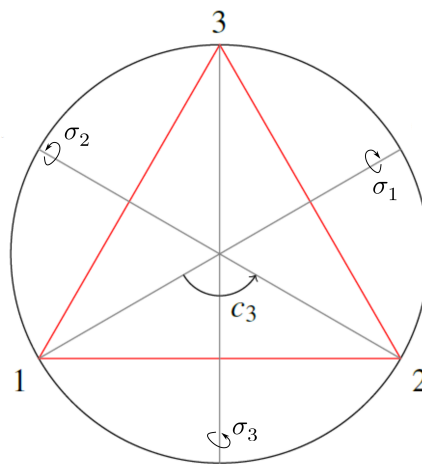


Symmetries in Physics

Seatwork 1

- (1) Terms of the day: Group homomorphism, the special unitary group $SU(n, \mathbb{C})$
- (2) Consider a regular (equilateral) triangle with corners x_1, x_2, x_3 in the Euclidean plane $\mathbb{R}^2$, and let $C_{3v} = \{e, c_3, c_3^2, \sigma_1, \sigma_2, \sigma_3\}$ be its symmetry group (with respect to $O(2)$), where c_3 is the rotation by 120° around the center O of the triangle, and σ_i is the reflection across the line through x_i and O .



- (a) Fill out the group multiplication table of C_{3v} : one places $g \cdot h$ in the cell where g is the row-label and h is the column-label. An example has been given ($\sigma_1 \cdot \sigma_2 = c_3$).

	e	c_3	c_3^2	σ_1	σ_2	σ_3
e						
c_3						
c_3^2						
σ_1					c_3	
σ_2						
σ_3						

Hint: Interpret the group elements of C_{3v} as permutations of the corners!

- (b) Determine all subgroups of C_{3v} . What do you notice about the respective sizes of the subgroups?

Recall: Subgroups are closed under the group operations (i.e., taking products and taking inverses).

(3) Always, sometimes, never:

Decide whether the given statements are always/sometimes/never true.

- (a) Subgroups of non-Abelian groups are non-Abelian.
- (b) Let $\mathcal{O}$ be a matrix group over $\mathbb{C}$. Then the determinant is a homomorphism from $\mathcal{O}$ to $(\mathbb{C} \setminus \{0\}, \cdot)$.
- (c) Given a finite Abelian group $(G, \cdot)$, there exists a finite non-Abelian group $(G', \cdot)$ with the same size as G such that $G \simeq G'$.
- (d) The n -cycle $(1\ 2\ \cdots\ n)$ can be expressed as a product of $(n - 1)$ 2-cycles.

*If you need extra paper, please ask the tutor.