Dr. Neil Manibo

Fakultät für Mathematik,
Universität Bielefeld

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Symmetries in Physics

Seatwork 2

(1) Terms of the day: quotient group G/H, stabiliser subgroup $\mathrm{Stab}_G(a)$ of an element $a \in \mathcal{O}$

(2) Let $H = \left\{ R_\theta = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} : \theta \in [0, 2\pi) \right\} \simeq \mathrm{SO}(2)$, seen as a subgroup of $\mathrm{SO}(3)$. This consists of all rotations around the z-axis. In this exercise, we would like to show that H is **not** a normal subgroup. To do this, it suffices to show the following:

- there exists $g \in \mathrm{SO}(3)$ and $h \in H$, such that

$$ghg^{-1} \notin \mathrm{SO}(2).$$

(a) Let $g_\theta \in \mathrm{SO}(3)$ be the rotation around the x-axis with the angle θ. Determine g_θ in matrix form with respect to the canonical basis vectors.

(b) Determine $(g_\theta)^{-1}$.
Hint: Remember that $g_\theta \in \mathrm{SO}(3)$!

(c) Show that $g_\theta h g_\theta^{-1} \notin \mathrm{SO}(2)$, for some $h \in H$.
Hint: It suffices to show that the product is not of the form R_θ.

*If you need extra paper, please ask the tutor.