

Symmetries in Physics

Exercise Sheet 2

- (4) (a) Let $H < G$ be a subgroup of G , and let $G/H = \{gH : g \in G\}$ be the corresponding factor set, consisting of the right cosets of H . Define a product on G/H by

$$gH \cdot kH = (gk)H.$$

Show that if this product is independent of the choice of representatives $g, k \in G$, then H is normal in G .

- (b) Show that the set $H \subset G$ of symmetries of $a \in \mathcal{O}$ given in Definition 1.18 is in fact a subgroup of G .

(2+2 points)

- (5) The group $O(2)$ consists of rotations r_α and reflections m_α , which act on

$$S^1 = \{z \in \mathbb{C} : |z| = 1\}$$

via

$$r_\alpha(e^{i\varphi}) = e^{i(\varphi+\alpha)}, \quad m_\alpha(e^{i\varphi}) = e^{i(2\alpha-\varphi)}.$$

- (a) Express each of the products $r_\alpha m_\beta$, $m_\beta r_\alpha$ and $m_\alpha m_\beta$ as a single rotation or reflection.
- (b) Express each of the conjugate elements $r_\alpha^{-1} m_\beta r_\alpha$, $m_\beta^{-1} r_\alpha m_\beta$ and $m_\beta^{-1} m_\alpha m_\beta$ as a single rotation or reflection.

(2+2 points)

- (6) **AI exercise:** Just as the symmetries of the triangle in $O(2)$ form a subgroup (C_{3v}), so do the symmetries of a regular n -gon in the plane. Given $n \geq 3$, the group is called the dihedral group. Have a conversation with AI about dihedral groups and their normal subgroups, and consolidate your interaction (min. 2 prompts, max 2 pages A4) in a one-paragraph (min 4, max 8 sentences) summary. In the summary, make a short comment about the difference between the cases $n = 5$ and $n = 8$.

(Conversation: 2 points)

(Summary: 3 points)

