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Symmetries in Physics

Exercise Sheet 3

(7) The permutation $p = (1\,2\cdots n)$ generates a so called *cyclic group* of order n, which consists of all powers of p. A cyclic group C_n of order n with generator g is the Abelian group

$$C_n = \langle g \rangle = \{g^j : 0 \leqslant j \leqslant n-1\}.$$

Here, $g^0 = g^n = e$. The rotation subgroup of C_{3v} is isomorphic with C_3.

Under what conditions is the direct product $C_n \times C_m$ of two cyclic groups again cyclic?
Hint: For $C_n \times C_m$ to be cyclic, it suffices to find an element of order nm.

(3 points)

(8) The alternating group A_n on n elements is given by

$$A_n = \{\sigma \in S_n : \sigma \text{ is an even permutation}\}.$$

(a) The centraliser $C_{A_4}((1\,2\,3))$ of $(1\,2\,3)$ in A_4 is the group generated by $(1\,2\,3)$. Use this to show that the conjugacy class $[(123)]$ contains 4 elements.

(b) Construct the conjugacy table for A_4.
Hint: The conjugacy table for S_4 in the lecture notes and part (a) provide good starting points.

In physics/chemistry, the *rotational tetrahedral group* T is isomorphic with A_4.

(1+3 points)

(9) **AI exercise**: There are 9 non-Abelian groups of order 16 up to isomorphism. Have a conversation with AI, pick two out of the 9, and discuss with it why these two are different. Consolidate your interaction (min. 2 prompts, max 2 pages A4) in a one-paragraph (min 4, max 8 sentences) summary.

(Conversation: 2 points)
(Summary: 2 points)

Due at 12:00 on Wednesday, 04.11.2026