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Symmetries in Physics

Exercise Sheet 4

(10) (a) Let $\widehat{n} \in S^2$ an let $\underline{s} = \frac{1}{2}[\sigma_1, \sigma_3, \sigma_3]$ be as the in the lecture. Determine $(\widehat{n} \cdot \underline{s})^m$, for all $m \in \mathbb{N}$.

(b) Using (a), show that with the parametrisation $x_0 = \cos(\frac{\varphi}{2})$, $x_k = -\widehat{n}_k \cdot \sin(\frac{\varphi}{2})$, $\widehat{n} \in S^2$, the matrix $A \in \mathrm{SU}(2)$ can be written in the form

$$A(\widehat{n}, \varphi) = \exp(-i(\widehat{n} \cdot \underline{s})\varphi).$$

Hint: Express the right hand-side as a Taylor series.

(2+3 Points)

(11) **AI exercise**: In classical mechanics, *canonical transformations* on

$$(q_1, q_2, \ldots, q_n, p_1, p_2, \ldots p_n)$$

are coordinate changes that leave the underlying dynamics invariant. The subclass of such transformations which are *linear* on the $2n$ coordinates form a group. Have a conversation with AI about this group and find similarities with the orthogonal group $\mathrm{O}(n)$. Consolidate your interaction (min. 2 prompts, max 2 pages A4) in a one-paragraph (min 4, max 8 sentences) summary.

(Conversation: 2 points)
(Summary: 2 points)

Due at 12:00 on Wednesday, 11.11.2026