Dr. Neil Manibo

Fakultät für Mathematik,
Universität Bielefeld

Wintersemester 2026/27
Bielefeld, 11.11.2026

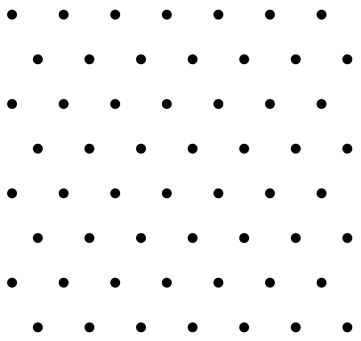
Symmetries in Physics

Exercise Sheet 4

(12) Let $G < E(2)$ be the full symmetry group (holohedry) of a hexagonal lattice in the plane. G is a *plane* group, a two-dimensional analogue of a space group.

(a) Determine the point group P and the translation subgroup T of G. Express the point group as a matrix group with respect to a basis of the translation lattice.

(b) Show that an element of G has finite order if and only if it has a fixed point.



(3+3 Points)

(13) Let $G = S_n$. Consider the map $\phi\colon G \to \mathrm{GL}(n, \mathbb{R})$ given by $\tau \mapsto (\delta_{i,\tau(j)})_{i,j}$. Show that ϕ is a homomorphism. What does $\mathrm{tr}(\phi(\tau))$ encode?

(2 Points)

(14) **AI exercise**: There are 14 possible classes of point groups in Theorem 2.9. Ask AI for an example of a molecule (with at least four atoms) having any of these point groups as stabiliser subgroup. Determine the number of conjugacy classes of the point group. Consolidate your interaction (min. 2 prompts, max 2 pages A4) in a one-paragraph (min 4, max 8 sentences) summary.

(Conversation: 2 points)
(Summary: 3 points)

Due at 12:00 on Wednesday, 18.11.2026