UNIVERSITÄT
BIELEFELD

$$M = \mathbb{1} + A, \qquad A = \begin{pmatrix} -\alpha & \alpha \\ \beta & -\beta \end{pmatrix}, \quad \alpha, \beta \geq 0, \quad \alpha + \beta < 1$$

$$\log(M) = \log(\mathbb{1} + A)$$

$$= \sum_{m=1}^{\infty} (-1)^{m+1} \frac{A^m}{m} \qquad (\text{da } \varrho = \alpha + \beta < 1)$$

$$= \sum_{m=1}^{\infty} (-1)^{m+1} \frac{1}{m} (-1)^{m-1} (\alpha+\beta)^{m-1} \cdot A$$

$$(\text{da } A^m = (-1)^{m-1} (\alpha+\beta)^{m-1} A)$$

$$= \underbrace{\sum_{m=1}^{\infty} (-1)^{m+1} \frac{(-(\alpha+\beta))^{m-1}}{m}}_{} A \qquad (\text{Ann.: } \alpha+\beta > 0)$$

$$= \frac{-1}{\alpha+\beta} \underbrace{\sum_{m=1}^{\infty} (-1)^{m+1} \frac{(-(\alpha+\beta))^{m}}{m}}_{}$$

$$= \log(1 - (\alpha+\beta))$$

$$= \underbrace{- \frac{\log(1-\alpha-\beta)}{\alpha+\beta}}_{>0} \cdot A \qquad \circledast$$

Für $\alpha+\beta = 0$ ist $\alpha = \beta = 0$ (wegen $\alpha, \beta \geq 0$) und $A = 0$, also $\log(\mathbb{1} + 0) = 0$, was auch aus $\circledast$ per l'Hospital herauskommt.