Heat Transfer in a Coffee Cup with Comsol Multiphysics

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1. Introduction and Mathematical Setting

2. Model Wizard

Start Comsol Multiphysics.
To start Comsol Multiphysics 5.0 open the Terminal and enter
• comsol -ckl

Model Wizard.
• In the New window, click Model Wizard.
• In the Model Wizard window, click 3D in the Select Space Dimension menu.
• In the Select Physics tree, select Mathematics>PDE Interfaces>Coefficient Form PDE (c).
• Click Add.
• Next, locate the Dependent Variables section.
• In the Field name text field, type T.
• In the Dependent Variables text field, type also T.
• Next, locate the Units section.
• From the Dependent variable quantity list, choose Temperature (K).
• From the Source term quantity list, choose Heat source (W/m³).
• Click Study.
• In the Select Study tree, select Preset Studies>Time Dependent.
• Click Done.

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Unit System.

- In the Model Builder window, click Untitled.mph (root).
- In the Settings window for Root, locate the Unit System section.
- From the Unit System list, choose SI.

Some Advanced Settings.

Hint: In the Model Builder window you should click on the Show icon and enable everything that is possible from the menu: Expand Sections (Equation View, Override and Contribution, Discretization, Stabilization, Advanced Physics Options, Advanced Study Options and Advanced Results Options). Done this, click Expand All icon.

3. Parameters and Variables

Parameters.

- In the Model Builder window, expand the Global node, right-click Definitions and select Parameters. (Alternatively: On the Model toolbar, click Parameters.)
- In the Settings window for Parameters, locate the Parameters section.
- In the table, enter the following settings:

<table>
<thead>
<tr>
<th>Name</th>
<th>Expression</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>0.073[m]</td>
<td>0.073 m</td>
<td>innerer Durchmesser der Tasse</td>
</tr>
<tr>
<td>L2</td>
<td>0.079[m]</td>
<td>0.079 m</td>
<td>äußerer Durchmesser der Tasse</td>
</tr>
<tr>
<td>L3</td>
<td>0.055[m]</td>
<td>0.055 m</td>
<td>Größe des Haltegriffs (bzw. Durchmesser des Toruses)</td>
</tr>
<tr>
<td>L4</td>
<td>0.007[m]</td>
<td>0.007 m</td>
<td>Dicke (bzw. innerer Durchmesser) des Haltegriffs</td>
</tr>
<tr>
<td>H1</td>
<td>0.08[m]</td>
<td>0.08 m</td>
<td>max. Füllhöhe der Tasse</td>
</tr>
<tr>
<td>H2</td>
<td>0.008[m]</td>
<td>0.008 m</td>
<td>Höhe des Bodens der Tasse</td>
</tr>
<tr>
<td>H3</td>
<td>0.07[m]</td>
<td>0.07 m</td>
<td>tatsächliche Füllhöhe</td>
</tr>
<tr>
<td>H4</td>
<td>0.046[m]</td>
<td>0.046 m</td>
<td>Abstand mittlerer Teil des Haltegriffes vom Tassenboden</td>
</tr>
</tbody>
</table>

Variables.

- In the Model Builder window, expand the Component 1 (comp1) node, right-click Definition and select Variables.
- In the Settings window for Variables, locate the Variables section.
- In the table, enter the following settings:
<table>
<thead>
<tr>
<th>Name</th>
<th>Expression</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>T0_wasser</td>
<td>(273.15+60)[K]</td>
<td>K</td>
<td>Temp. der Flüssigkeit zu Meßbeginn</td>
</tr>
<tr>
<td>T0_tasse</td>
<td>(273.15+23)[K]</td>
<td>K</td>
<td>Temp. der Tasse zu Meßbeginn</td>
</tr>
<tr>
<td>T_luft</td>
<td>(273.15+23)[K]</td>
<td>K</td>
<td>Temp. der Raumluft zu Meßbeginn</td>
</tr>
<tr>
<td>kappa_luft</td>
<td>10[W/(m²·K)]</td>
<td>W/(m²·K)</td>
<td>Wärmeübergangskoeffizient Luft</td>
</tr>
<tr>
<td>rho_wasser</td>
<td>995[kg/m³]</td>
<td>kg/m³</td>
<td>Dichte Luft (bei 30 °C)</td>
</tr>
<tr>
<td>c_wasser</td>
<td>4182[J/(kg·K)]</td>
<td>J/(kg·K)</td>
<td>spez. Wärmekapazität Wasser (20 °C)</td>
</tr>
<tr>
<td>lambda_wasser</td>
<td>0.597[W/(m·K)]</td>
<td>W/(m·K)</td>
<td>spez. Wärmeleitfähigkeit Wasser (20 °C)</td>
</tr>
<tr>
<td>rho_glas</td>
<td>2500[kg/m³]</td>
<td>kg/m³</td>
<td>Dichte Glas</td>
</tr>
<tr>
<td>c_glas</td>
<td>600[J/(kg·K)]</td>
<td>J/(kg·K)</td>
<td>spez. Wärmekapazität Glas</td>
</tr>
<tr>
<td>lambda_glas</td>
<td>0.76[W/(m·K)]</td>
<td>W/(m·K)</td>
<td>spez. Wärmeleitfähigkeit Glas</td>
</tr>
<tr>
<td>rho_porzellan</td>
<td>2300[kg/m³]</td>
<td>kg/m³</td>
<td>Dichte Porzellan</td>
</tr>
<tr>
<td>c_porzellan</td>
<td>730[J/(kg·K)]</td>
<td>J/(kg·K)</td>
<td>spez. Wärmeleitfähigkeit Porzellan</td>
</tr>
<tr>
<td>lambda_porzellan</td>
<td>1.03[W/(m·K)]</td>
<td>W/(m·K)</td>
<td>spez. Wärmeleitfähigkeit Porzellan</td>
</tr>
<tr>
<td>g_wasser</td>
<td>0[m²/s³]</td>
<td>m²/s³</td>
<td>Wärmeleitfähigkeit Porzellan</td>
</tr>
<tr>
<td>g_glas</td>
<td>0[m²/s³]</td>
<td>m²/s³</td>
<td></td>
</tr>
<tr>
<td>g_porzellan</td>
<td>0[m²/s³]</td>
<td>m²/s³</td>
<td></td>
</tr>
</tbody>
</table>

4. **Geometry**

**Work Plane 1 (middle part of the coffee cup).**

- In the **Model Builder** window, expand the **Component 1 (comp1)** node, right-click **Geometry** and select **Work Plane**.
- In the **Settings** window for Work Plane, locate the **Plane Definition** section.
- In the *-coordinate* text field, type H2.
- In the **Model Builder** window, expand the **Component 1 (comp1)>Geometry 1>Work Plane 1 (wp1)** node, right-click **Plane Geometry** and select **Circle**.
- In the **Settings** window for Circle 1, locate the **Size and Shape** section.
- In the **Radius** text field, type L2/2.
- Right-click **Plane Geometry** again and select **Circle**.
- In the **Settings** window for Circle 2, locate the **Size and Shape** section.
- In the **Radius** text field, type L1/2.
- Right-click **Plane Geometry** once more and select **Booleans and Partitions>Difference**.
- In the **Settings** window for Difference, locate the **Difference** section.
- In the subsection **Objects to add**, the active button should be activated (green). Now, switching to the **Graphics** window and left-click on the larger circle. In the **Objects to add** list, the entry c1 now appears.
- Next, activate the activ button in the subsection **Objects to substract**. Switching once more to the **Graphics** window and left-click on the smaller (inner) circle. In the **Objects to substract** list, the entry c2 now appears.
- In the **Model Builder** window, right-click on **Work Plane 1 (wp1)** and select **Extrude**.
- In the **Settings** window for Extrude, locate the **Distance from Plane** section.
- In the **Distances (m)** table substitute 1 by H3.
- In the **Model Builder** window, right-click on **Extrude 1 (ext1)** and select **Build Selected** to check the result.
Work Plane 2 (top of the coffee cup).
- In the Model Builder window, expand the Component 1 (comp1) node, right-click Geometry and select Work Plane.
- In the Settings window for Work Plane, locate the Plane Definition section.
- In the Model Builder window, expand the Component 1 (comp1)>Geometry 1>Work Plane 2 (wp2) node, right-click Plane Geometry and select Circle.
- In the Settings window for Circle 1, locate the Size and Shape section.
- In the Radius text field, type L2/2.
- Right-click Plane Geometry again and select Circle.
- In the Settings window for Circle 2, locate the Size and Shape section.
- In the Radius text field, type L1/2.
- Right-click Plane Geometry once more and select Booleans and Partions>Difference.
- In the subsection Objects to add, the active button should be activated (green). Now, switching to the Graphics window and left-click on the larger circle. In the Objects to add list, the entry c1 now appears.
- Next, activate the active button in the subsection Objects to subtract. Switching once more to the Graphics window and left-click on the smaller (inner) circle. In the Objects to subtract list, the entry c2 now appears.
- In the Model Builder window, right-click on Work Plane 2 (wp2) and select Extrude.
- In the Settings window for Extrude, locate the Distance from Plane section.
- In the Distances (m) table substitute 1 by H1-H3.
- In the Model Builder window, right-click on Extrude 2 (ext2) and select Build Selected to check the result.

Cylinder (bottom of the coffee cup).
- In the Model Builder window, right-click on Geometry 1 and select Cylinder.
- In the Settings window for Cylinder, locate the Size and Shape section.
- In the Radius text field, type L2/2 and in the Height text field H2.
- In the Model Builder window, right-click on Cylinder 1 (cyl1) and select Build Selected to check the result.

Torus (handle of the cup).
- In the Model Builder window, right-click on Geometry 1 and select More Primitives>Torus.
- In the Settings window for Cylinder, locate the Size and Shape section.
- In the Major radius text field, type L3/2, in the Minor radius text field L4/2 and in the Revolution angle text field 180.
- Next, locate the Position section.
- In the x text field, type -L2/2+0.001, in the y text field 0 and in the z text field H4.
- Finally, locate the Axis section.
- As Axis type choose y-axis.
• In the **Model Builder** window, right-click on **Torus 1 (tor1)** and select **Build Selected** to check the result.

**Union.**

• In the **Model Builder** window, right-click on **Geometry 1** and select **Booleans and Partitions>Union**.

• In the **Settings** window for Union, locate the **Union** section.

• In the subsection **Input objects**, the active button should be activated (green). Now, switching to the **Graphics** window and left-click on the top, the middle, the bottom and on the handle of the cup. In the **Input objects** list, the entries **ext1, ext2, cyl1** and **tor1** now appears.

**Cylinder (liquid in the cup).**

• In the **Model Builder** window, right-click on **Geometry 1** and select **Cylinder**.

• In the **Settings** window for Cylinder 2, locate the **Size and Shape** section.

• In the **Radius** text field, type **L1/2** and in the **Height** text field **H3**.

• Next, locate the **Position** section.

• In the **x** text field, type **0**, in the **y** text field **0** and in the **z** text field **H2**.

• In the **Model Builder** window, right-click on **Form Union (fin)** and select **Build Selected** to check the overall result.

• The complete geometry is shown in Figure 4.1.

![Abbildung 4.1. Geometry (spatial domain) of the coffee cup that is filled with a fluid (coffee)](image)

**5. The Heat Equation**

**Partial differential equation.**

• In the **Model Builder** window, expand the **Component 1 (comp 1)>Coefficient Form PDE (c)** node, then click **Coefficient Form PDE 1**.

• We are now in **Settings** window for Coefficient Form PDE.
• In the **Diffusion Coefficient** \( c \) text field, type \( \lambda_{\text{glas}} \).
• In the **Source Term** \( f \) text field, type \( \rho_{\text{glas}} g_{\text{glas}} \).
• In the **Damping or Mass Coefficient** \( d \) text field, type \( \rho_{\text{glas}} c_{\text{glas}} \).
• In the **Model Builder** window, right-click **Coefficient For PDE (c)** and select **Coefficient Form PDE**.
• In the **Model Builder** window, click **Coefficient Form PDE 2**.
• In the **Settings** window for Coefficient Form PDE 2, locate the **Domain Selection** section.
• As **Selection** choose **Manual**. The domain list should contain only domain 7, that corresponds to the liquid.
• In the **Diffusion Coefficient** \( c \) text field, type \( \lambda_{\text{wasser}} \).
• In the **Source Term** \( f \) text field, type \( \rho_{\text{wasser}} g_{\text{wasser}} \).
• In the **Damping or Mass Coefficient** \( d \) text field, type \( \rho_{\text{wasser}} c_{\text{wasser}} \).

**Boundary conditions.**
• In the **Model Builder** window, right-click **Coefficient For PDE (c)** and select **Flux/Source**.
• In the **Model Builder** window, click **Flux/Source 1**.
• In the **Settings** window for Flux/Source 1, locate the **Boundary Selection** section.
• As **Selection** choose **Manual**. The boundary list should contain the boundaries 1-13,27,28,30,36-41,43,45-48. These correspond to all faces that have contact with the air.
• In the **Boundary Flux/Source** \( g \) text field, type \(-\kappa_{\text{luft}}(T-T_{\text{luft}})\).

**Initial values.**
• In the **Model Builder** window, select **Coefficient For PDE (c)>Initial Values 1**.
• In the **Settings** window for Initial Values 1, locate the **Initial Values** section.
• For **Initial value for T** specify \( T_{0\text{-tasse}} \).
• In the **Model Builder** window, right-click **Coefficient For PDE (c)** and select **Initial Values**.
• In the **Settings** window for Initial Values 2, locate the **Domain Selection** section.
• As **Selection** choose **Manual**. The domain list should contain only domain 7, that corresponds to the liquid.
• In the **Settings** window for Initial Values 2, locate the **Initial Values** section.
• For **Initial value for T** specify \( T_{0\text{-wasser}} \).
• The initial temperature distribution \( T(x,0) = T_0(x) \) at initial time \( t = 0 \) is shown in Figure 5.1.

**Spatial discretization.**
• In the **Model Builder** window, click **Coefficient Form PDE (c)**.
• In the **Settings** window for Coefficient Form PDE, locate the **Discretization** section.
• As **Shape function type** select **Lagrange** and as **Element order** choose **Linear**.

6. **Mesh, Study Settings and Computation**

**Generating Mesh.**
• In the **Model Builder** window, click **Mesh 1**.
Abbildung 5.1. Initial temperature distribution $T$ (in °C) of the coffee cup at initial time $t = 0$ (in s)

- In the Settings window for Mesh, locate the Mesh Settings section.
- As Sequence type select User-controlled mesh.
- In the Model Builder window, expand the Mesh 1 node, then click Size.
- In the Settings window for Size, locate the Element Size Parameters section.
- In the Maximum element size text field, type 0.011.

Study Settings.
- In the Model Builder window, expand the Study 1 node, then click Step 1: Time Dependent.
- In the Settings window for Time Dependent, locate the Study Settings section.
- In the Times text field, type range(0,0.1,60).
- Enable the checkbox for Relative Tolerance.
- In the Model Builder window, right-click Solver Configurations and select Show Default Solver.
- In the Model Builder window, expand the Study 1>Solver Configurations>Time-Dependent Solver 1 node and click Time-Dependent Solver 1.
- In the Settings for Time-Dependent Solver 1, locate the Absolute Tolerance section.
- As Global method choose unscaled.
- In the Settings for Time-Dependent Solver 1, now locate the Time Stepping section.
- As Method choose BDF, as Steps taken by solver choose Intermediate and as Maximum BDF oder choose 2.
- In the Model Builder window, expand the Study 1>Solver Configurations>Time-Dependent Solver 1 node and click Fully Coupled 1.
- In the Setting for Fully Coupled 1, locate the Method and Termination section.
- As Nonlinear method choose Automatic (Newton).
Computation.

- In the Model Builder window, expand the Study 1 node, right-click on Step 1: Time-Dependent and select Compute Selected Step.

7. Postprocessing and Graphical Output

Surface Plot.

- In the Model Builder window, expand the Results node, then right-click Results>3D Plot Group 1 and select Surface from the menu.
- Next, right-click on Results>3D Plot Group 1>Slice 1 and select Delete from the menu.
- Now, click on Results>3D Plot Group 1.
- In the Settings window for 3D Plot Group 1, first locate the Data section. For Data set select Study 1/Solution 1 and for Time (s) select 60.
- Next, locate the Plot Settings section. Disable the check box for Plot data set edges.
- In the Model Builder window, click on Results>3D Plot Group 1>Surface 1.
- In the Settings window for Surface 1, first locate the Data section and select From Parent for Data set.
- Next, locate the Expression section. For Expression type T, for Unit select degC from the list and disable the check box for Description.
- Now, locate the Title section. For Title type select custom. In the Type and data subsection disable the check boxes for Type, Description and Expression, and enable the check box for Unit. In the text field for Prefix type Temperatur T.
- Finally, locate the Range section. Enable the check boxes for Manual color range and Manual data range. In the text fields for Minimum type 22 and for Maximum type 61.
- The result is shown in Figure 7.1.

Abbildung 7.1. Surface of the heat distribution (temperature) $T$ (in °C) of the coffee cup at time $t = 60$ (in s)
Slice Plot.

- In the **Model Builder** window, right-click on **Results** and select **3D Plot Group** from the menu.
- Next, right-click on **Results>3D Plot Group 2** and select **Slice** from the menu.
- Now, click **Results>3D Plot Group 2**.
- In the **Settings** window for 3D Plot Group 2, locate the **Data** section. As **Data set** select **Study 1/Solution 1** and as **Time (s)** select 60 from the list.
- In the **Model Builder** window, click **Results>3D Plot Group 2>Slice 1**.
- In the **Settings** window for Slice 1, first locate the **Data** section and select **From Parent** for **Data set**.
- Next, locate the **Expression** section. For **Expression** type T, for **Unit** select degC from the list and **disable** the check box for **Description**.
- Now, locate the **Title** section. For **Title type** select custom. In the **Type and data** subsection disable the check boxes for **Type**, **Description** and **Expression**, and **enable** the check box for **Unit**. In the text field for **Prefix** type Temperatur T.
- Next, locate the **Plane Data** section, select **xy-plane** for **Plane** and type 4 in the **Planes** text field.
- Finally, locate the **Range** section. **Enable** the check boxes for **Manual color range** and **Manual data range**. In the text fields for **Minimum** type 22 and for **Maximum** type 61.
- The result is shown in Figure 7.2.

![Slice Plot Example](image.png)

**Abbildung 7.2.** Slices of the heat distribution (temperature) $T$ (in °C) of the coffee cup at time $t = 60$ (in s)

Animation.

- In the **Model Builder** window, right-click on **Results>Export** and select **Animation** from the menu.
- Click on **Results>Export>Animation 1**.
• In the **Settings** window for Animation 1, first locate the **Scene** section and select **3D Plot Group 1** for **Subject**.

• Next, locate the **Output** section. As **Output type** select **Movie**, as **Format** select GIF and in the **File name** text field type **coffee.gif**. In the **Frames per second** text field type **10**.

• Now, locate the **Frames** section. As **Frame selection** select **Number of frames** and in the **Number of frames** text field type **100**. In the **Width** text field type **800** and in the **Height** text field **600**.

• Finally, locate the **Layout** section and enable all check boxes, i.e. for **Include**, **Title**, **Color legend**, **Grid**, **Axis orientation** and **Logotype**. In the **Font size** text field type **10**.

• Now, click on the **Export (F8)** symbol to create the movie. The symbol can be found in the header of the **Settings** window.

8. **Save the Model**

**Save File.**

• Select **File>Save As...**

• Select a desired folder, where the model should be saved, and enter **CoffeeCup.mph** as the **Name** for the model.

• Click **OK**.