Dynamic Patterns in PDEs

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July 4, 2013

Abstract: In this talk we consider systems of reaction diffusion equations

$$\begin{aligned} u_t(x,t) &= A\triangle u(x,t) + f(u(x,t)),\ t>0,\ x\in\mathbb{R}^d,\ d\geqslant 2, \\ u(x,0) &= u_0(x) \qquad\qquad\qquad\quad ,\ t=0,\ x\in\mathbb{R}^d. \end{aligned} \tag{1}$$

with N by N diffusion matrix A, smooth nonlinearity $f:\mathbb{R}^N\to\mathbb{R}^N$, initial data $u_0:\mathbb{R}^d\to\mathbb{R}^N$ and solution $u:\mathbb{R}^d\times[0,\infty[\to\mathbb{R}^N$. We are particularly interested in the analysis and numerical computation of special dynamic patterns such as traveling and rotating waves. These solutions have a fixed shape and travel or rotate at constant velocity. We discuss the freezing method which enables us to determine seperately the profile and the velocity of the emerging patterns.
Special asymptotic states of (1) are rotating waves of the form $u(x,t)=v(e^{-tS}x)$, where S is a d by d skew-symmetric matrix and v is the profile which satisfies the elliptic system

$$A\triangle v(x) + \langle Sx, \nabla v(x)\rangle + f(v(x)) = 0,\ x\in\mathbb{R}^d,\ d\geqslant 2.$$

For such systems we prove under certain conditions that every classical solution which falls below a certain threshold at infinity, must decay exponentially in space. Furthermore, we characterize the point spectra and essential spectra for linearizations about rotating waves. As example we discuss spinning solitons that arise from the quintic-cubic complex Ginzburg-Landau equation. This study is motivated by the stability problem for rotating waves in several variables.