QE "Optimization", WS 2017/18

Problem Set No. 12

Submit your solutions by **4.12.2017**.
The problems will be discussed in the tutorials.

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1. [**6 Points**] Using the Lagrange multiplier method, solve the constrained optimization problem in $\mathbb{R}^2$

$$\max \;/\; \min \quad f(x,y) = xy$$
$$\text{subject to the constraint } \; x^2 + y^2 = 1.$$

- i Explain why you expect the problem to be solvable;
- ii Show that the constrained Qualification fails only at the point $x = y = 0$;
- iii Form the Lagrangean and write the 1st order conditions;
- iv Find all 4 candidates for a solution of the constrained optimization problem;
- v Choose the global max/min among them.

Answer: the max/min values of f are $-1/2$ and $1/2$.

2. [**7 points**] Using the Lagrange multiplier method, solve the constrained optimization problem in $\mathbb{R}^3$

$$\max / \min \quad f(x,y,z) = x^2 + y^2 + z$$
$$\text{subject to} \quad (x-1)^2 + y^2 = 5, \; y = z.$$

- i Explain why you expect the problem to be solvable;
- ii Simplify the problem by reducing to the case of 2 variables;
- iii Check that the Constrained Qualification holds at all points obeying the equality constraint;

iv Form the Lagrangean and write the 1st order conditions;

v Find all candidates for a solution and show that the max/min values of f are 11 and 1 respectively.

3. [**6 Points**] Consider the problem

$$\begin{array}{rc} \max & f(x,y) = 2x + 3y \\ \text{subject to} & \sqrt{x} + \sqrt{y} = 5. \end{array}$$

Show that the Langrange multiplier method suggests the wrong solution $(x, y) = (9, 4)$. Compare $f(9, 4)$ and $f(25, 0)$. Explain why the Lagrange theorem cannot be used here to find the solution.

4. [**6 Points**] Find the local extrema of the function

$$\begin{array}{c} f(x,y) = x + 2y \\ \text{subject to} \quad x^2 + y^2 = 5. \end{array}$$

Apply the necessary and **sufficient** conditions of the 1st order.

5. [**6 Points**] Using the Lagrange multiplier method, solve the constrained optimization problem in $\mathbb{R}^3$

$$\begin{array}{rc} \max / \min & f(x,y,z) = x + y + z \\ \text{subject to} & x^2 + y^2 + z^2 = 12. \end{array}$$

Apply necessary and **sufficient** conditions of the 1st order.