QE "Optimization", WS 2017/18

Problem Set No. 7

Submit your solutions by **30.10.2017**, 12 noon.
The problems will be discussed in the tutorials.

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1. [**6 Points**] Show from the definition (the existence of the limit in Definition 2.1.1) that the following functions $f\colon \mathbb{R}^2 \to \mathbb{R}$ are partially differentiable with respect to both x and y at all points $(x, y) \in \mathbb{R}^2$, where f is given by

$$\begin{aligned} \textbf{(a)}\ f(x,y) &= 2x + y \\ \textbf{(b)}\ f(x,y) &= x^2 + y^2. \end{aligned}$$

Also from the definition, calculate all partial derivatives at the point $(2, 3)$.

2. [**6 Points**] Calculate the gradient of the functions

$$\begin{aligned} \textbf{(a)}\ f(x,y,z) &= x^2 + ze^{2y},\ (x,y,z) \in \mathbb{R}^3, \\ \textbf{(b)}\ g(x,y,z) &= e^{xyz},\ (x,y,z) \in \mathbb{R}^3. \end{aligned}$$

3. [**3 Points**] Calculate the directional derivative of the function $f(x, y) = \sin(xy)$ at point $(1, 0)$ along direction $v = \big(1/2, \sqrt{3}/2\big)$.

4. [**4 Points**] Calculate the directional derivative of the function g from Problem **2(b)** at point $(1, 1, 1)$ along direction $v = \Big(1/\sqrt{6}, \sqrt{2/3}, -1/\sqrt{6}\Big)$.

5. [**3 Points**] Calculate the directional derivative of the function

$$f(x, y, z) = x^3 + 2ze^{3y}, \quad (x, y, z) \in \mathbb{R}^3,$$

at point $(-1, 0, 1)$ along direction $v = \big(1/2, -1/2, 1/\sqrt{2}\big)$.

6. [**6 Points**] Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x,y) := \begin{cases} \frac{y^3}{\sqrt{x^2+y^2}}, & \text{if } (x,y) \neq 0. \\ 0, & \text{if } (x,y) = 0. \end{cases}$$

Show that f is totally differentiable at $(0, 0)$. *Hint:* Apply Theorem 2.3.