QE "Optimization", WS 2017/18

Problem Set No. 8

Submit your solutions by **6.11.2017**, 12 noon.
The problems will be discussed in the tutorials.

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1. [**4 Points; see also Lecture Notes**] Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \left(\frac{3xy^2}{4x^2+4y^4}\right)^2, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Show that:
(i) f is continuous on every line drawn through $(0,0)$;
(ii) f is not continuous at $(0,0)$.
[**Hint:** Consider $x_k := y_k^2$ with $y_k \to 0$ as $k \to \infty$.)
2. [**4 Points**] Define the function F of two variables $(x,y) \in \mathbb{R}^2$ by

$$F(x,y) = f(g(x,y), h(x,y)),$$

where

$$f(s,t) = st^2,\ g(x,y) = x + y^2,\ h(x,y) = x^2 y.$$

Use the chain rule to find $\partial_x F(x,y)$ and $\partial_y F(x,y)$.
3. [**4 Points**] The amount x of some good demanded depends on the price p of the good and the amount a the producer spends on advertising: $x = f(p,a)$, with $\partial_p f(p,a) < 0$ and $\partial_a f(p,a) > 0$ for all (p,a). The price depends on the weather, measured by the parameter w, and the tax rate t: $p = g(w,t)$, where $\partial_w g(w,t) > 0$ and $\partial_t g(w,t) < 0$ for all (w,t). The amount of advertising depends only on t: $a = h(t)$, with $h'(t) > 0$. If the tax rate increases does the demand for the good necessarily increase or necessarily decrease, or neither?
4. [**4 Points**] Calculate the second order Taylor polynomial for the Cobb-Douglas function with a given parameter $a \in (0,1)$

$$f(x,y) := x^a y^{1-a}, \quad U = \{(x,y) \in \mathbb{R}^2 \mid x > 0, y > 0\},$$

at the point $(1, 1)$ (without the remainder term).

5. [**4 Points**] Calculate the second order Taylor polynomial for the function

$$f(x, y, z) := xe^{-y} + y + z + 1, \quad (x, y, z) \in \mathbb{R}^3,$$

at point $(1, 0, 0)$ (without the remainder term).