QE "Optimization", WS 2017/18

Problem Set No. 9

Submit your solutions by **13.11.2017**.
The problems will be discussed in the tutorials.

---

1. [**4 Points**] Consider the function

$$F(x, y, z) := x^2 - y^2 + z^3.$$

(a) If $x = 6$ and $y = 3$, find z which satisfies $F(x, y, z) = 0$.

(b) Does this equation define z as an implicit function of x, y near $x = 6$, $y = 3$?

(c) If so, compute $\partial z/\partial x(6, 3)$ and $\partial z/\partial y(6, 3)$.

2. [**4 Points**] Consider the function

$$F(x, y, z) = x^4 + 2x \cos y + \sin z.$$

Show that the equation $F(x, y, z) = 0$ defines z as an implicit function of x, y near $x = y = z = 0$. Compute $\partial z/\partial x$ and $\partial z/\partial y$ at this point.

3. [**10 Points**] Consider the equation

$$x^3 + 3y^2 + 4xz^2 - 3z^2 y = 1.$$

Does this equation define z as a function of x, y:

(a) in the neighbourhood of $x = 1$, $y = 1$?

(b) in the neighbourhood of $x = 1$, $y = 0$?

(c) in the neighbourhood of $x = 1/2$, $y = 0$?

If so, compute $\partial z/\partial x$ and $\partial z/\partial y$ at these points.

4. [**5 Points**] Check that the system of equations

$$\begin{cases} u + xe^y + v = e - 1 \\ x + e^{u+v^2} - y = e^{-1} \end{cases}$$

defines $u = u(x,y)$ and $v = v(x,y)$ as differentiable functions of x, y around the point $(x,y,u,v) = (1,1,-1,0)$. Differentiate the system and find the values of u'_x, u'_y, v'_x, v'_y at this point.

5. [**4 Points**] Show that the map

$$f(x,y) := \big(x + e^y, y + e^{-x}\big)$$

is everywhere locally invertible. Calculate $Df^{-1}(x,y)$ at $x = 1$, $y = -1$.

6. [**6 Points**] Show that at each $(x,y) \in \mathbb{R}^2$ the map

$$f(x,y) := (e^x \cos y, e^x \sin y)$$

is locally invertible but is not globally invertible. Calculate $Df^{-1}(x,y)$ at $x = 0$, $y = 0$.