Pólya Urns: Rates of Convergence

(Andrea Kuntschik)

In 1923, G. Pólya and F. Eggenberger introduced an urn model to describe series of interlinked events modelling an effect known as "the rich get richer". Later on, this urn model has been generalised, comprehensively studied and referred to as the "generalised Pólya-Eggenberger urn schemeör, simply, the "Pólya urn scheme". Pólya urns play an important role in probability theory and its applications, e.g., computer science. Turning the spotlight on balanced, irreducible and non-deterministic Pólya urn schemes with two colours, we will have a closer look on the number of balls of a specific colour. This quantity is known to be, properly scaled, subject to central limit theorems. Depending on the ratio of the eigenvalues of the replacement matrix, there are two regimes of limit laws: normal weak limits and non-normal weak limits. We determine rates of convergence in both regimes: It turns out that the rates are driven by the ratio of the eigenvalues, as well. Our approach is based on a recursive description of the evolution of the urn process via embedding into a random discrete tree structure revealing some sort of self-similarity. Eventually, this self-similarity is exploited to apply strategies derived from the contraction method.