Computational Peridynamics

Mini-Workshop: Mathematical Analysis for Peridynamics
Mathematisches Forschungsinstitut Oberwolfach

January 16-22, 2011

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Computing Research Center
Sandia National Laboratories
Albuquerque, New Mexico

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Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy's National Nuclear Security Administration under contract DE-AC04-94AL85000.

What is Peridynamics?

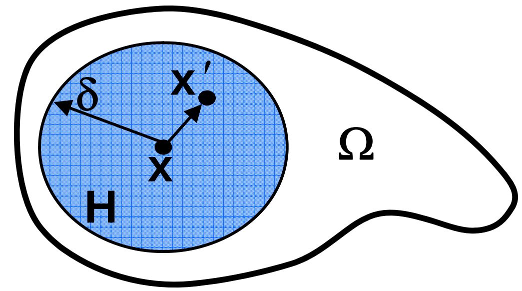
- **Peridynamics is a nonlocal extension of classical solid mechanics that permits discontinuous solutions**

"In peridynamics, cracks are part of the solution, not part of the problem."
- F. Bobaru

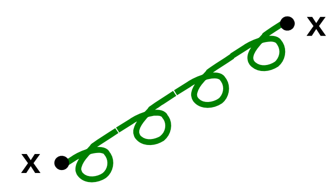
- **Peridynamic equation of motion (integral, nonlocal)**

$$\rho \ddot{u}(x,t) = \int_{H} f(u' - u, x' - x)\,dV' + b(x,t)$$

  - Replace PDEs with integral equations
  - Utilize same equation everywhere; nothing "special" about cracks
  - No assumption of differentiable fields (admits fracture)
  - When bonds stretch too much, they break
  - No obstacle to integrating nonsmooth functions
  - f(·, ·) is "force" function; contains constitutive model
  - f = 0 for particles x,x' more than δ apart (like cutoff radius in MD!)
  - PD is "continuum form of molecular dynamics"



Peridynamic Domain



Peridynamic "bond"

- **Impact**
  - Nonlocality
  - Larger solution space (fracture)
  - Account for material behavior at small & large length scales **(multiscale material model)**

- **Ancestors**
  - Kröner, Eringen, Edelen, Kunin, Rogula, etc.

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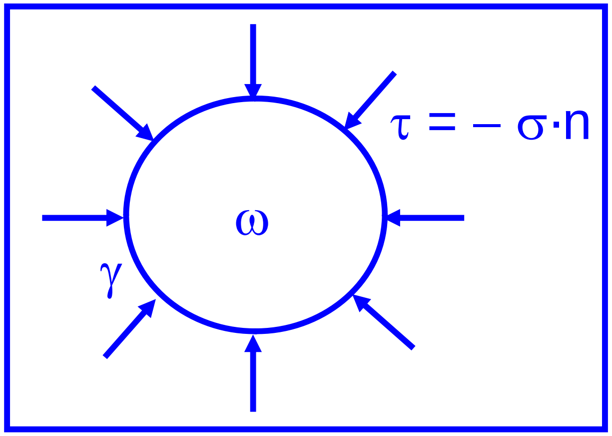
Local vs. Nonlocal Models

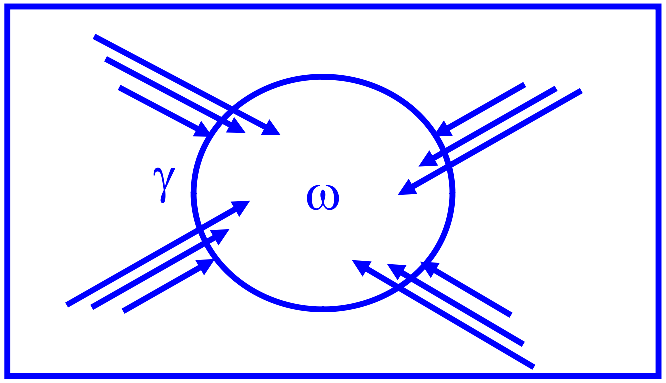
"It can be said that all physical phenomena are nonlocal. Locality is a fiction invented by idealists."
- A. Cemal Eringen

- **Local model:**
  - Contact force
  - Exterior of circle imparts force to interior via surface
  - Cauchy cut principle (free body diagram)
- Examples:
  - Classical elasticity, etc.
  - Any PDE-based model



Local Domains

- **Nonlocal model:**
  - Action-at-a-distance
  - **Exterior of circle imparts force to interior – not just at surface**
- Examples:
  - Molecular dynamics
  - Peridynamics

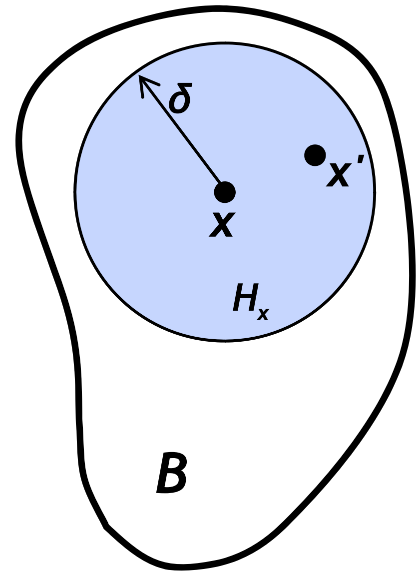


Nonlocal Domains

- **Foreshadowing**
  - Algorithms and numerical methods for nonlocal models are fundamentally different (and generally more expensive!) than local (classical) models.

Length Scales

- What does it mean to have a length scale?
  - What does it mean to be multiscale?
- Example #1: $\ddot{u}(x) = au''(x)$
  - Equation has no length scale; same dynamics at all scales
- Example #2: $\ddot{u}(x) = au''(x) + bu''''(x)$
  - Dimensional analysis gives that sqrt(b/a) has units of length
  - Rescaling x can make first term dominant or second term dominant
  - Scaling of x changes behavior of equation
- Peridynamic horizon δ represents a *length scale*
  - Behavior (dynamics) of EOM vary with length scale
  - Exhibit desired physics on applied length scale
- Peridynamics provides desired dynamics at multiple length scales!
  - Rescaling space (equivalent to rescaling δ) provides transition from microscale to macroscale (classical) models!
- Connection between nonlocal models and higher-gradient models



Peridynamic Model (nonlocal)

$$\rho\ddot{u}(x,t) = \int_{-\delta}^{\delta} \frac{c}{|\varepsilon|}\left[u(x+\varepsilon,t) - u(x,t)\right]d\varepsilon$$

Taylor series

Higher-Gradient Model (weakly nonlocal)

$$\rho\ddot{u}(x,t) = K_a\left[\frac{d^2u}{dx^2} + \frac{\delta^2}{24}\frac{d^4u}{dx^4} + \frac{\delta^4}{1080}\frac{d^6u}{dx^6} + \ldots\right]$$

lim $\delta \to 0$

Local, Scale Invariant

$$\rho\ddot{u}(x,t) = K_a\frac{d^2u}{dx^2}$$

Part I

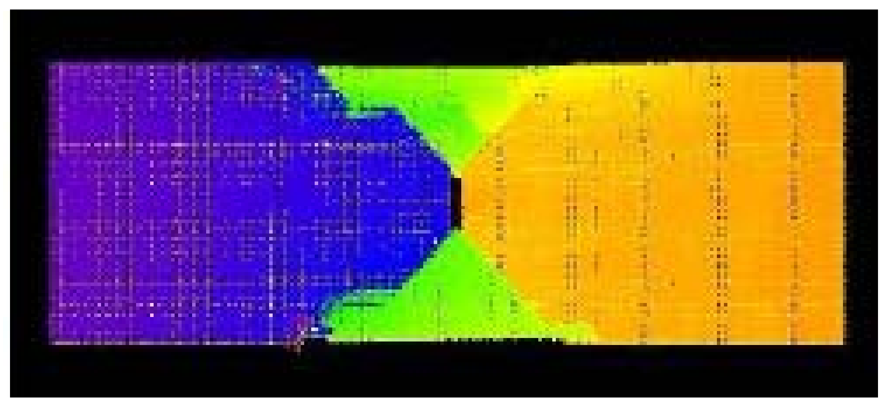
Applications and Codes

Simulation performed with EMU Fortran 90 code (Silling)

Some Applications…

- Splitting and fracture mode changes in fiber-reinforced composites*
- Fiber orientation between plies strongly influences crack growth

Typical crack growth in notched laminate (photo courtesy Boeing)



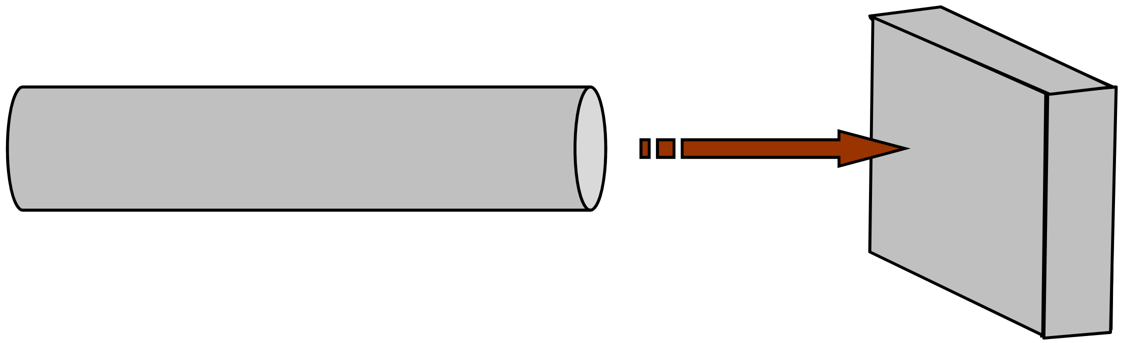
Peridynamic Model

* E. Askari, F. Bobaru, R.B. Lehoucq, M.L. Parks, S.A. Silling, O.Weckner, Peridynamics for multiscale materials modeling, in SciDAC 2008, Seattle, Washington, vol. 125 of Journal of Physics: Conference Series, (012078) 2008.

Simulation performed with EMU Fortran 90 code (Silling)

Some Applications…

- Taylor impact test of 6061-T6 aluminum*



Experiment

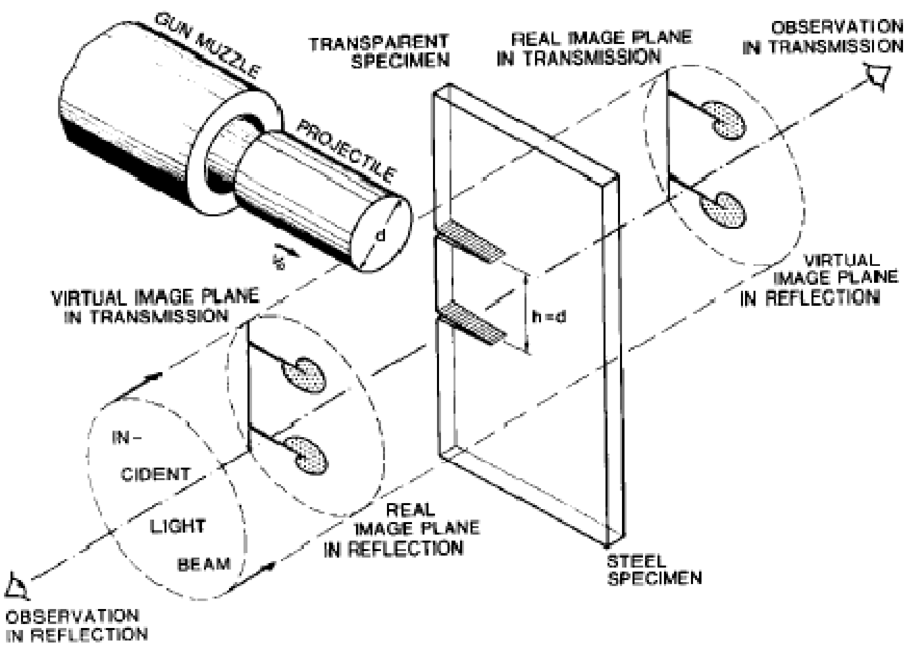
Peridynamic Model*

* J. Foster, S.A. Silling, W.W. Chen, Viscoplasticity Using Peridynamics, Sandia National Laboratories Technical Report SAND2008-7835, 2008.

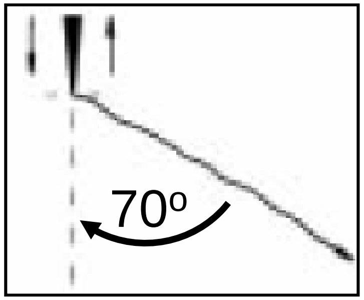
Simulation performed with EMU Fortran 90 code (Silling)

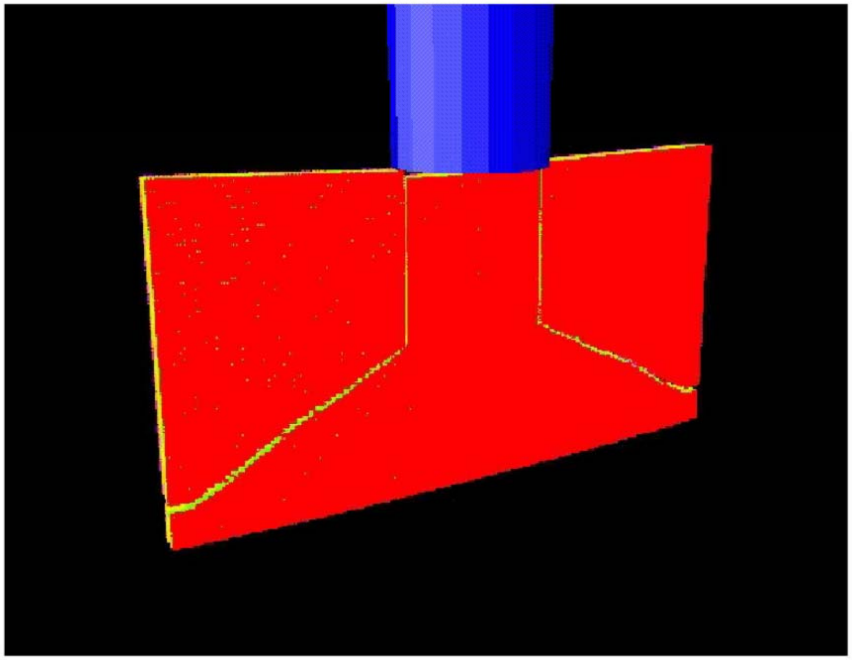
Some Applications…

- Dynamic fracture in steel (Kalthoff & Winkler, 1988)
- Mode-II loading at notch tips results in mode-I cracks at 70º angle
- Peridynamic model reproduces the 70º crack angle*



Experimental Results





Peridynamic Model

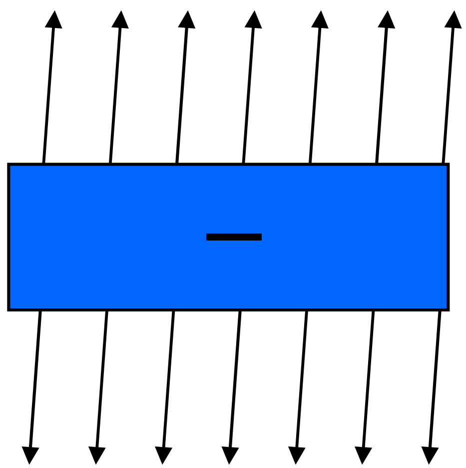
* S. A. Silling, Dynamic fracture modeling with a meshfree peridynamic code, in Computational Fluid and Solid Mechanics 2003, K.J. Bathe, ed., Elsevier, pp. 641-644.

Simulation performed with EMU Fortran 90 code (Silling)

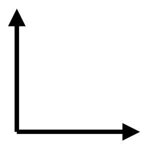
Some Applications…

- Discrete peridynamic model exhibits mesh-independent crack growth
- Plate with a pre-existing defect is subjected to prescribed boundary velocities
- Crack growth direction depends continuously on loading direction

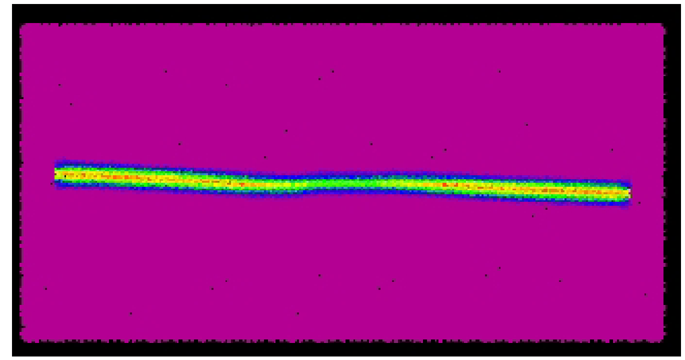
$$\dot{\varepsilon} = (0.25\mathrm{s}^{-1}) \begin{bmatrix} 0 & 0.1 \\ 0 & 1 \end{bmatrix}$$



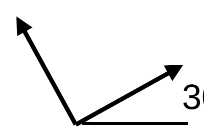
(Mostly) Mode-I loading
(with a little Mode-II)



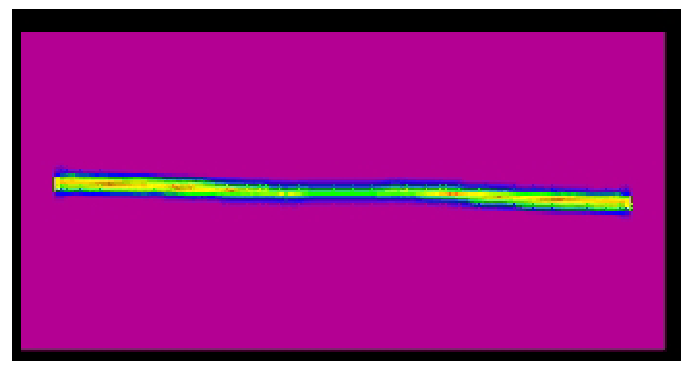
Original grid direction



Damage



Rotated grid direction



- Nonlocal network of bonds in many directions allows cracks to grow in any direction.

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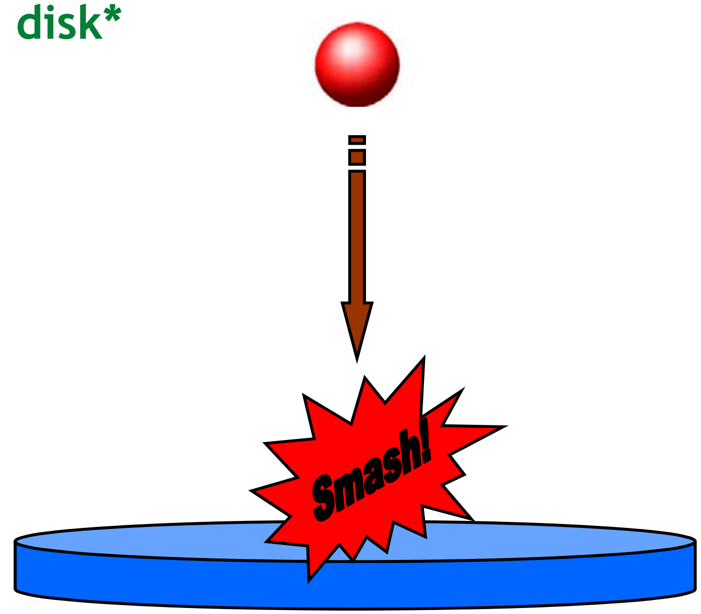
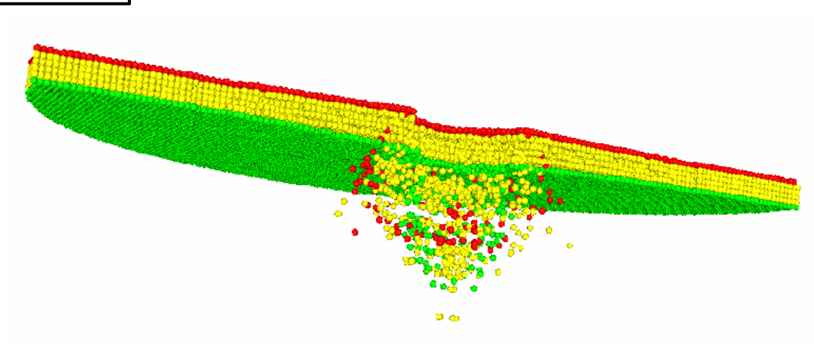
Simulation performed with PDLAMMPS code (Parks)

Some Applications…

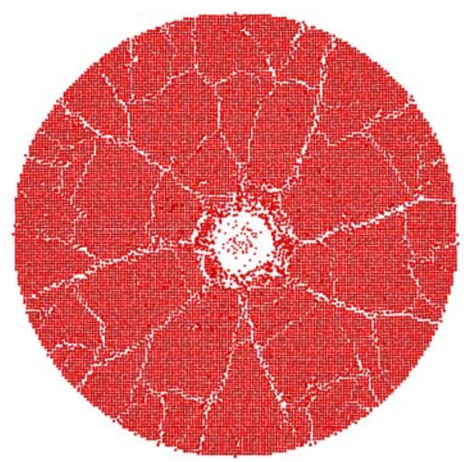
- **Example Simulation:** Hard sphere impact on brittle disk*
- Spherical Projectile
  - Diameter: 0.01 m
  - Velocity: 100 m/s
- Target Disk
  - Diameter: 0.074 m,
  - Thickness: 0.0025 m
  - Elastic modulus: 14.9 Gpa
  - Density: 2200 kg/m^3
- Discretization
  - Mesh spacing: 0.005 m
  - 100,000 particles
  - Simulation time: 0.2 milliseconds



Results



Side View



Top Monolayer

*S.A. Silling and E. Askari, *A meshfree method based on the peridynamic model of solid mechanics,* Comp. and Struct., 83, pp. 1526-1535, 2005.

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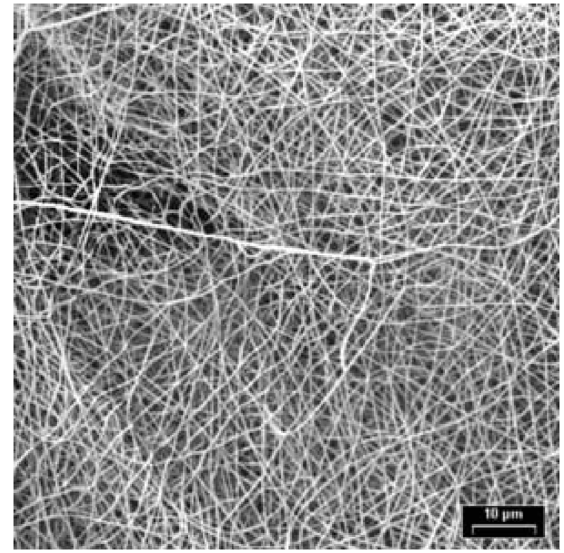
Simulation performed with PDLAMMPS code (Parks)

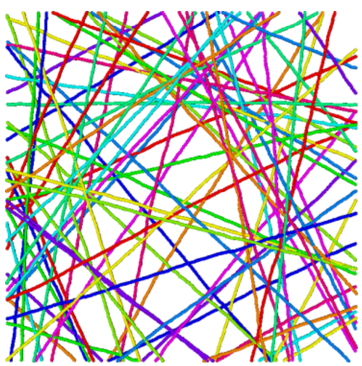
Some Applications…

- **Example Simulation:** Failure of Nanofiber Network*
- Nanofiber networks
  - Large surface area to volume ratio
  - High axial strength and extreme flexibility
  - Used in composites, protective clothing, catalysis, electronics, chemical warfare defense
- Numerical Model
  - 400 nm x 400 nm x 10 nm
  - Biaxial strain induces failure
  - PD PMB material model (augmented for van der Walls forces)
- Findings**
  - van der Walls important for strength and toughness
  - Heterogeneity in bonds strength increases toughness, ductility

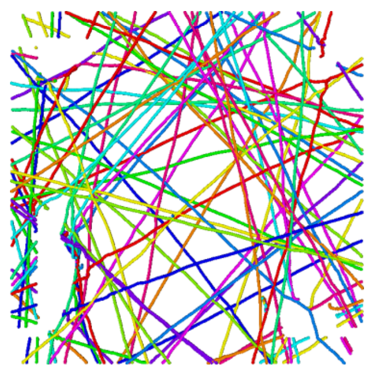


Nanofiber Network
(http://www.me.wpi.edu/MTE/current_projects.htm)

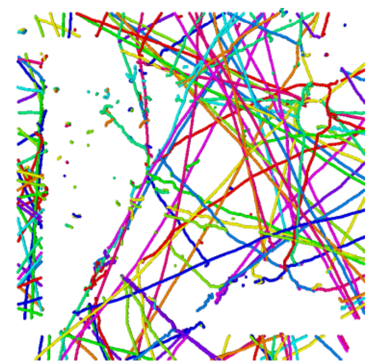
Results



t=0; 0% strain



t=30 ns; 17.6% strain



t=50 ns; 29.4% strain

* E. Askari, F. Bobaru, R.B. Lehoucq, M.L. Parks, S.A. Silling, and O. Weckner, Peridynamics for multiscale materials modeling, in SciDAC 2008, Seattle, Washington, July 13-17, 2008, vol. 125 of Journal of Physics: Conference Series, (012078) 2008.
** F. Bobaru, Influence of van der Waals forces on increasing the strength and toughness in dynamic fracture of nanofiber networks: a peridynamic approach, Modelling Simul. Mater. Sci. Eng., 15 (2007), pp. 397-417.

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Simulation performed with PDLAMMPS code (Parks)

Some Applications…

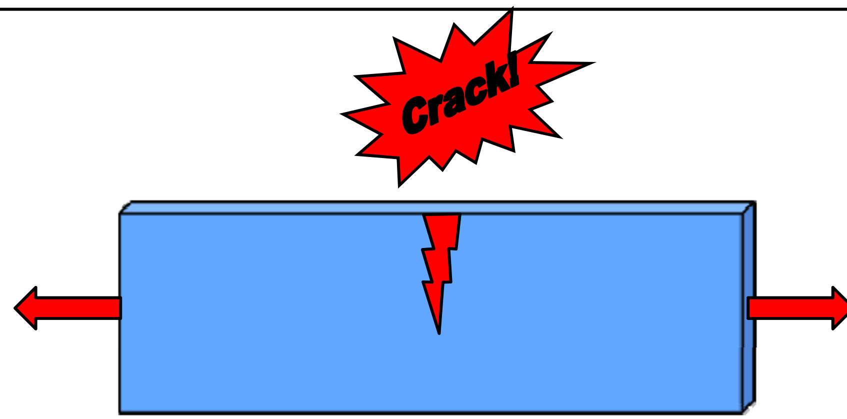
- **Example simulation:** Dynamic brittle fracture in glass
  - Joint with Florin Bobaru, Youn-Doh Ha (Nebraska), & Stewart Silling (SNL)

- Soda-lime glass plate (microscope slide)
  - Dimensions: 3” x 1” x 0.05”
  - Density: 2.44 g/cm3
  - Elastic Modulus: 79.0 Gpa

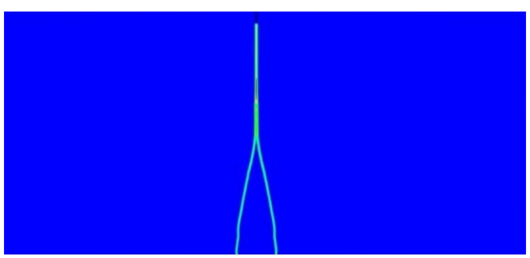
- Discretization (finest)
  - Mesh spacing: 35 microns
  - Approx. 82 million particles
  - Time: 50 microseconds (20k timesteps)

Setup

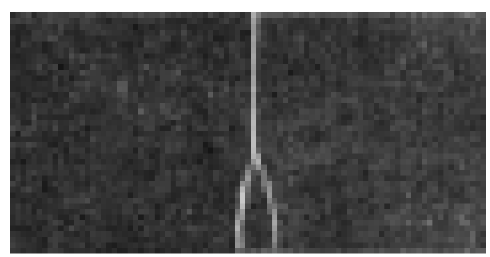
- Glass microscope slide
- Dimensions: 3” x 1” x 0.05”
- Notch at top, pull on ends



Results



Peridynamics



Physical Experiment*

Strain Energy Density

*S F. Bowden, J. Brunton, J. Field, and A. Heyes, *Controlled fracture of brittle solids and interruption of electrical current*, Nature, 216, 42, pp.38-42, 1967.

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Simulation performed with PDLAMMPS code (Parks)

Some Applications…

- Dawn (LLNL): IBM BG/P System
  - 500 teraflops; 147,456 cores
- Part of Sequoia procurement
  - 20 petaflops; 1.6 million cores
- Discretization (finest)
  - Mesh spacing: 35 microns
  - Approx. 82 million particles
  - Time: 50 microseconds (20k timesteps)
  - 6 hours on 65k cores
- Largest peridynamic simulations in history

***Dawn* at LLNL**

Weak Scaling Results

# Cores	# Particles	Particles/Core	Runtime (sec)	T(P)/T(P=512)
512	262,144	4096	14.417	1.000
4,096	2,097,152	4096	14.708	0.980
32,768	16,777,216	4096	15.275	0.963

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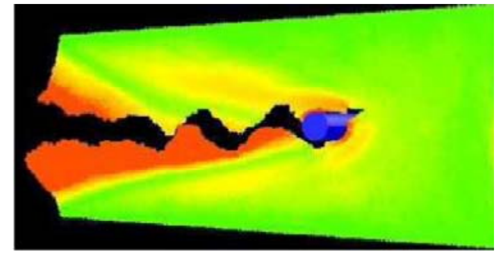
Some Codes…



Instability in slow tearing of elastic membrane*
(EMU)

- **EMU** (F90) EMU
  - First Peridynamic code
  - Research code
  - EMU has many features, but export controlled…
- **EMU variants** (F90)
  - Many developers have branched EMU and started their own development line
  - Kraken, etc.

- **PDLAMMPS (Peridynamics-in-LAMMPS)** (C++)
  - Core set of features, massively parallel

- **Peridigm** (C++) Peridigm
  - Production peridynamic code
  - Multiphysics
  - Component-based
  - Massively parallel
  - UQ/Optimization/Calibration, etc.
- **Peridynamics in SIERRA/SM (Presto)**
  - Utilizes Sandia's LAME material library

Fragmentation of metal ring
(Peridigm)

*S.A. Silling and F. Bobaru, "Peridynamic modeling of membranes and fibers", International Journal of Non-Linear Mechanics, 40(2-3): 395-409 (2005).

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Peridynamics-in-LAMMPS (PDLAMMPS)

- **Goals**
  - Provide open source peridynamic code (distributed with LAMMPS; lammps.sandia.gov)
  - Provide (nonlocal) continuum mechanics simulation capability within MD code
  - Leverage portability, fast parallel implementation of LAMMPS (Stand on the shoulders of LAMMPS developers)
- **Capability**
  - Prototype microelastic brittle (PMB), Linear peridynamic solid (LPS) models
  - Viscoplastic model
  - General boundary conditions
  - Material inhomogenity
  - LAMMPS highly extensible; easy to introduce new potentials and features
  - More information & user's guide at www.sandia.gov/~mlparks (Click on "software")
- **Papers**
  - M.L. Parks, P. Seleson, S.J. Plimpton, R.B. Lehoucq, and S.A. Silling, *Peridynamics with LAMMPS: A User Guide*, Sandia Tech Report SAND 2010-5549.
  - M.L. Parks, R.B. Lehoucq, S.J. Plimpton, and S.A. Silling, *Implementing Peridynamics within a molecular dynamics code*, Computer Physics Communications 179(11) pp. 777-783, 2008.

- ***A personal observation…***
  - Time from starting implementation to running first experiment: Two weeks
  - Time for same using XFEM, other approaches: ????
  - Conclusion: Peridynamics is an expedient approach for fracture modeling

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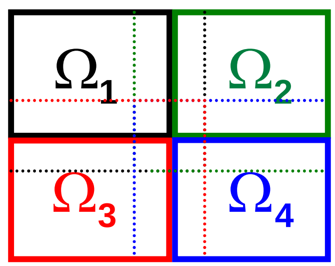
Peridynamics-in-LAMMPS (PDLAMMPS)

- **LAMMPS (Sandia's open source MD package)**
  - Large-scale Atomic/Molecular Massively Parallel Simulator
  - Open source, massively parallel, general purpose MD simulator
  - Many interatomic potentials for bio/polymers, solid state materials, etc.
  - Demonstrated scalability on Top500 computers (BlueGene/P, Red Storm)
  - Leverage MPI framework for particle model

- **MPI: spatial data decomposition + ghosting**



- **Added "SI" units to LAMMPS for macroscale simulations**
  - MD: angstroms, femtoseconds, etc.
  - PD: meters, seconds, etc.

Multiphysics Peridynamics via Agile Components

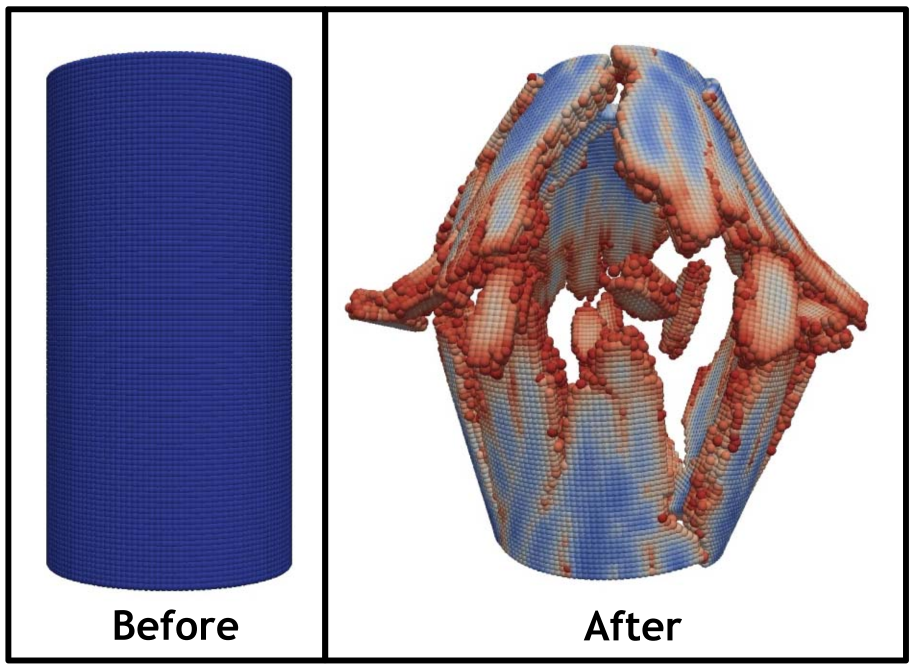
- ❑ **Agile components: World-class algorithms delivered as reusable libraries**
  - ❑ Full range of **independent** yet **interoperable** software components
  - ❑ Interfaces *and* capabilities
  - ❑ Choose capabilities a-la-carte (toolkit, not monolithic framework)
  - ❑ Software quality **tools** and **practices**
- ❑ **Rapid production strategic goals**
  - ❑ Enable rapid development of new production codes; Reduce redundancy
- ❑ **Prototype application: Peridigm**
  - ❑ Particle-based, not mesh based (like FEM)
  - ❑ Multi-physics
  - ❑ Scalable
  - ❑ Optimization-enabled
  - ❑ Born-in UQ
  - ❑ Interface with SIERRA mechanics
- ❑ **Collaborators**:
  - ❑ Dave Littlewood (1444)
  - ❑ Stewart Silling (1444)
  - ❑ John Mitchell (1444)
  - ❑ John Aidun (PM,1425)

Peridigm

Peridigm Planned FY11 Development

- Exodus reader (CUBIT)
- Multiple material blocks
- Implicit time integration
- Plasticity model
- Viscoelastic model
- UQ, calibration, etc. (DAKOTA)

Exploding Brittle Cylinder



Multiphysics Peridynamics via Agile Components

Peridigm

Software Quality Tools

- Mailing Lists
- Version Control
- Build System
- Testing (CTest)
- Project Management
- Issue Tracking
- Wiki

DAKOTA

- UQ
- Optimization
- Error Estimation
- Calibration

ParaView

- Visualization

Visualization Toolkit

- Service Tools

Trilinos

Parallelization Tools

- Data Structures (Epetra)
- Load Balancing (Zoltan)

Analysis Tools

- UQ (Stokhos)
- Optimization (MOOCHO)

Services

- Interfaces (Thyra)
- Tools (Teuchos, TriUtils)
- Field Manager (Phalanx)
- DAKOTA Interface (TriKota)

Solver Tools

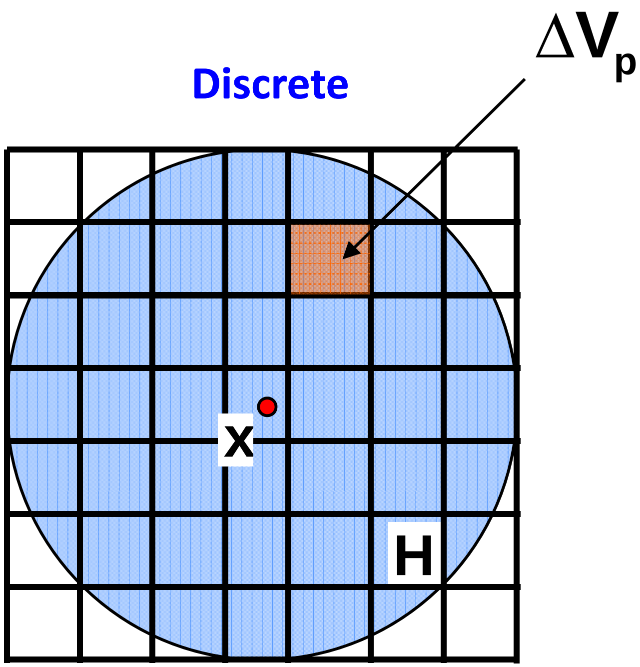
- Iterative Solvers (Belos)
- Direct Solvers (Amesos)
- Nonlinear Solvers (NOX)
- Eigensolvers (Anasazi)
- Preconditioners (IFPack)
- Multilevel (ML)

Part II

Discretizations and Numerical Methods

Discretizing Peridynamics

- **Spatial Discretization**
  - Approximate integral with sum*
  - Midpoint quadrature
  - Piecewise constant approximation



$$\sum_p f(u(x_p,t)-u(x_i,t),x_p-x_i)\Delta V_p$$

- **Temporal Discretization**
  - Explicit central difference in time

$$\ddot{u}(x,t) \approx \ddot{u}_i^n = \frac{u_i^{n+1} - 2u_i^n + u_i^{n-1}}{\Delta t^2}$$

  - Velocity-Verlet

$$v_i^{n+1/2} = v_i^n + \left(\frac{\Delta t}{2m}\right) f_i^n$$

$$u_i^{n+1} = u_i^n + (\Delta t) v_i^{n+1/2}$$

$$v_i^{n+1} = v_i^{n+1/2} + \left(\frac{\Delta t}{2m}\right) f_i^{n+1}$$

- **This approach is sometimes called the "EMU" numerical method (Silling)**

*S.A. Silling and E. Askari, *A meshfree method based on the peridynamic model of solid mechanics,* Computers and Structures, 83, pp. 1526-1535, 2005.

Discretizing Peridynamics

- **This approach is simple but expedient. What more can we do?**

- **Temporal discretization**
  - Implicit time integration (Newmark-beta method, etc.)

- **Spatial discretization (strong form)**
  - Midpoint quadrature (EMU method)
  - Gauss quadrature*

- **Spatial discretization (weak form)**
  - **Nonlocal Galerkin finite elements (1D)***
    - Nonlocal integration-by-parts*
    - Nonlocal mass & stiffness matrices, force vector*

- **Let's explore Peridynamic finite elements…**

*E. Emmrich and O. Weckner, *The peridynamic equation and its spatial discretization, Math. Model. Anal.*, 12(1), pp. 17-27, 2007.

Part III

Peridynamic Finite Elements*

*B. Aksoylu and M.L. Parks, *Variational Theory and Domain Decomposition for Nonlocal Problems*. Applied Mathematics and Computation. To Appear. 2011.

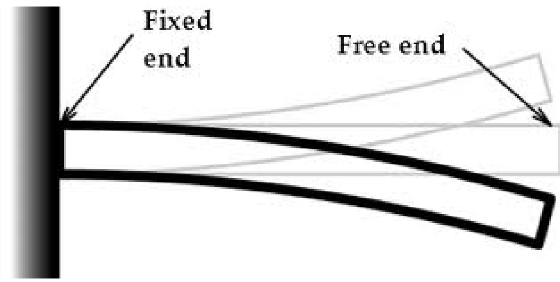
Why is Conditioning Important?

- **What is the condition number of a matrix?**

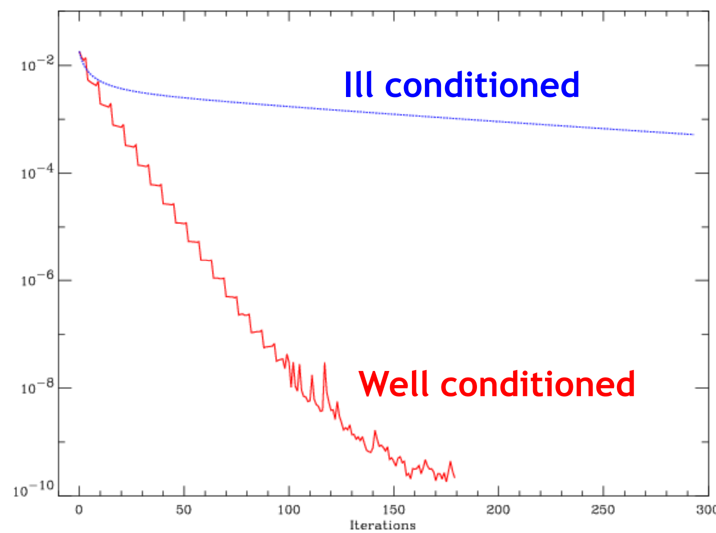
$$\kappa(A) = \|A\| \cdot \|A^{-1}\|$$

- **Why do we care?**
  - Condition number dictate convergence rates of linear solvers
  - Condition numbers dictate the accuracy of computed solution
  - Rule of thumb:
    If $\kappa(A) = 10^{16-d}$, then computed solution has d digits of accuracy.

    If $\kappa(A) = 10^{16}$, expect zero digits of accuracy!
- Old saying: ***"You get the answer you deserve…"***
- **Driving motivation for effective preconditioners**



Cantilevered beam



Convergence curves for optimal Krylov methods

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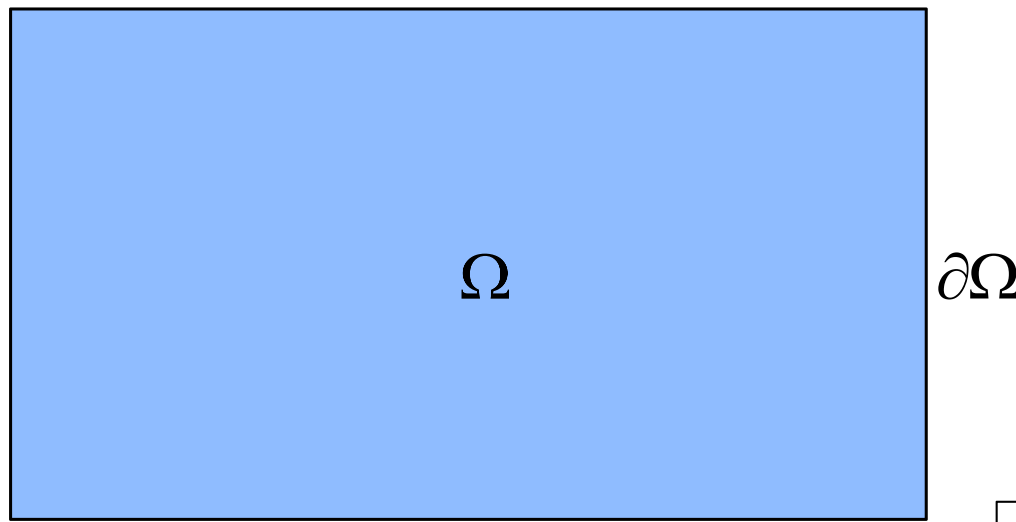
Why is Conditioning Important?

- **Why do I care about condition numbers of peridynamic models?**
  - First step towards scalable preconditioners
  - First step towards effective utilization of leadership class supercomputers for peridynamic simulations
- **New component in nonlocal modeling is peridynamic horizon δ**
  - How does δ affect the conditioning?
  - Develop preconditioners/solvers optimized for nonlocal models at extreme scales
- **DOE current computing platforms**
  - Jaguar (ORNL)
  - 2.595 petaflops (~2.5 quadrillion calculations per second)
  - 224,162 cores

- **DOE future computing platforms**
  - Exaflop machines by 2018

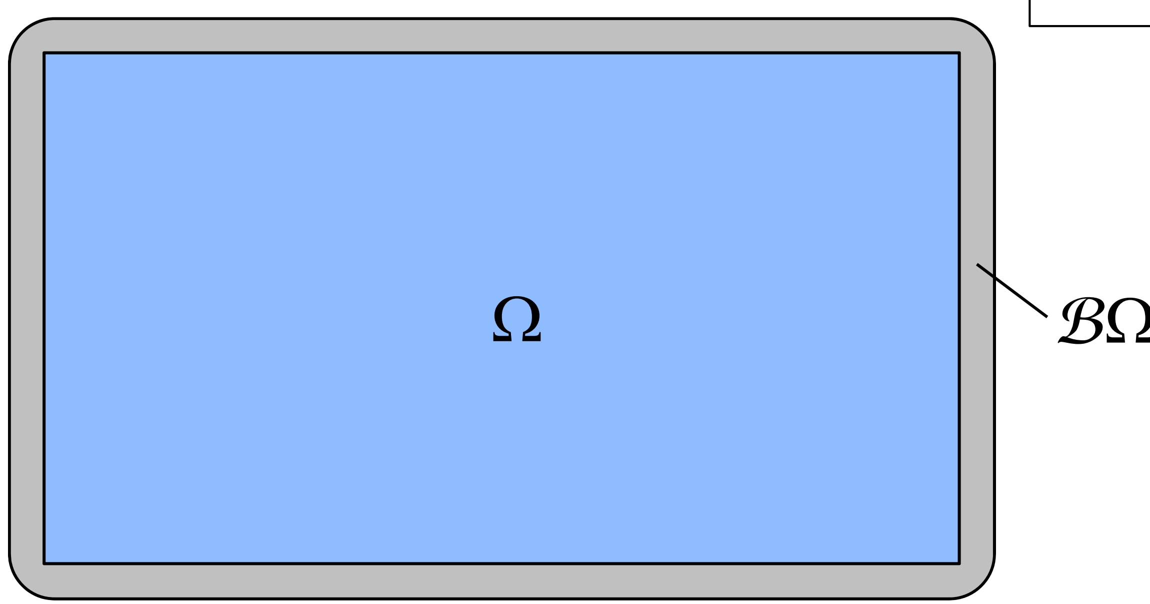
Nonlocal Boundaries

- Classical domain and boundary: $\overline{\Omega} = \Omega \cup \partial\Omega$



- Nonlocal domain and boundary: $\overline{\overline{\Omega}} = \Omega \cup \mathcal{B}\Omega$

$\partial\Omega$ interacts with all points in $\mathcal{B}\Omega$



Nonlocal Weak Form

- EMU/PDLAMMPS discretize strong form of equation (like finite differences)
- What about nonlocal finite elements?
- Prototype operator

$$\mathcal{L}\{u\}(x) = -\int_{\overline{\overline{\Omega}}} C(x,x')\,[u(x') - u(x)]\,dx'$$

$$C(x,x') = C(x',x)$$

$$C(x,x') = 0 \text{ if } \|x - x'\| > \delta$$

- Need nonlocal weak form* → Multiply by test function and "integrate by parts"

$$a(u,v) = -\int_{\overline{\overline{\Omega}}}\int_{\overline{\overline{\Omega}}} C(x,x')\,[u(x') - u(x)]\,v(x)\,dx'dx$$

$$= \frac{1}{2}\int_{\overline{\overline{\Omega}}}\int_{\overline{\overline{\Omega}}} C(x,x')\,[u(x') - u(x)]\,[v(x') - v(x)]\,dx'dx$$

- Compare with local Poisson operator

$$-\nabla^2 u(x) \Longrightarrow \frac{1}{2}\int \nabla u \cdot \nabla v\, dx$$

*E. Emmrich and O. Weckner, *The peridynamic equation and its spatial discretization, Math. Model. Anal.*, 12(1), pp. 17-27, 2007.

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Nonlocal Quadrature

- Review: Local Quadrature
  - One integral required
  - Compute products of ***gradients*** of shape functions and apply Gauss quadrature
  - Gradient ***drops*** polynomial order (lower order quadrature scheme required)

$$a(u,v) = \frac{1}{2}\int \nabla u \cdot \nabla v \, dx$$

- Nonlocal Quadrature
  - Two integrals required
  - Compute products of differences of shape functions and integrate
  - No gradient → higher polynomial order (higher order quadrature needed)
  - Nonlocality generates substantially more work over each element
  - Discontinuous integrands a challenge for quadrature routines (more later…)

$$a(u,v) = -\int_{\bar{\bar{\Omega}}}\int_{\bar{\bar{\Omega}}} C(x,x')\left[u(x') - u(x)\right] v(x)\,dx'dx$$

$$= \frac{1}{2}\int_{\bar{\bar{\Omega}}}\int_{\bar{\bar{\Omega}}} C(x,x')\left[u(x') - u(x)\right]\left[v(x') - v(x)\right] dx'dx$$

- Integration by parts is standard in local (classical) FEM.
- <u>Discussion</u>: **Does it serve any purpose here?**

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Spectral Equivalence

- For simplicity, assume

$$C(x,x') = \chi_\delta(x - x') \equiv \begin{cases} 1 & \text{if } \|x - x'\| \le \delta \\ 0 & \text{otherwise} \end{cases}$$

"Canonical" Kernel Function

- Principle Theorem*

$$\lambda_1(\bar{\bar{\Omega}})\delta^{d+2} \le \frac{a(u,u)}{\|u\|_{L_2(\bar{\bar{\Omega}})}} \le \lambda_2(\bar{\bar{\Omega}})\delta^{d} \qquad u \in L_{2,0}(\bar{\bar{\Omega}})$$

- Let K be a finite element discretization of a(u,u). Then,

$$\kappa(K) \sim \mathcal{O}(\delta^{-2})$$

- This is not tight!
  - Consider lim $\delta \to 0$. Cond # estimate $\to \infty$, true $\kappa(K) \to h^{-2}$.
  - Condition number not mesh independent (bound is mesh independent).
  - In practice, observe **very** weak mesh dependence.
  - Bound descriptive when $h < \delta$.
  - Alternative approach: Zhou & Du†
- Dominant length scale in nonlocal model set by δ.
  - Contrast with local model, where length scaled introduced by h

*B. Aksoylu and M.L. Parks, *Variational Theory and Domain Decomposition for Nonlocal Problems*. Applied Mathematics and Computation. To Appear. 2011.

† K. Zhou, Q. Du, Mathematical and numerical analysis of linear peridynamic models with nonlocal boundary conditions, SIAM J. Num. Anal., 48(5), pp. 1759–1780, 2010.

† Q. Du and K. Zhou. Mathematical analysis for the peridynamic nonlocal continuum theory. Mathematical Modelling and Numerical Analysis, 2010. doi:10.1051/m2an/2010040.

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Nonlocal Weak Form – 1D

- Let Ω = (0,1), $\mathcal{B}\Omega$ = [-δ,0]$\cup$[1, δ].
- u=0 on $\mathcal{B}\Omega$
- Let $C(x,x') = \begin{cases} 1 & \text{if } \|x - x'\| \le \delta \\ 0 & \text{otherwise} \end{cases}$
- Weak form becomes

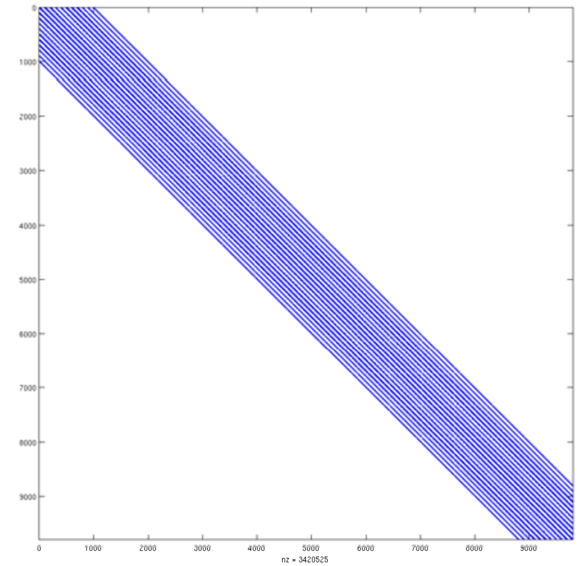
$$a(u,v) = -\int_{-\delta}^{\delta} \int_{x-\delta}^{x+\delta} \left[u(x') - u(x)\right] v(x) dx' dx$$

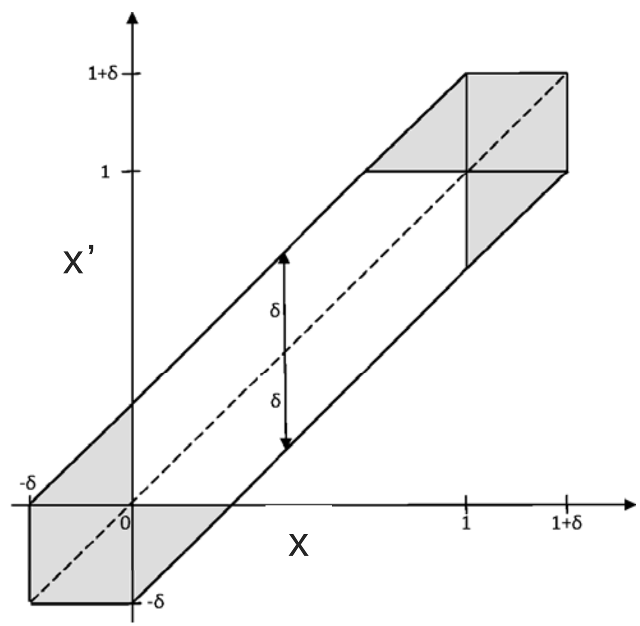
- Numerical Study
  - PW constant and PW linear SFs
  - Hold δ constant, vary h
  - Hold h constant, vary δ

Stiffness Matrix
Sparsity Pattern

2D Model

(10,000 unknowns,
3.4M nnz)





Integration
Domain in (x,x')

(grey = outside Ω)

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Nonlocal Finite Elements and Conditioning – 1D

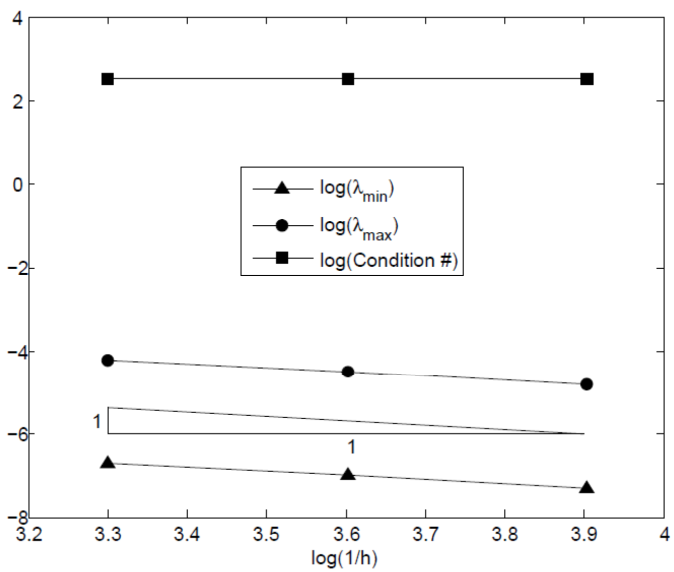
- Observations: $\kappa(K) \sim O(\delta^{-2})$, only weak h-dependence

(a) Constant δ, vary h.

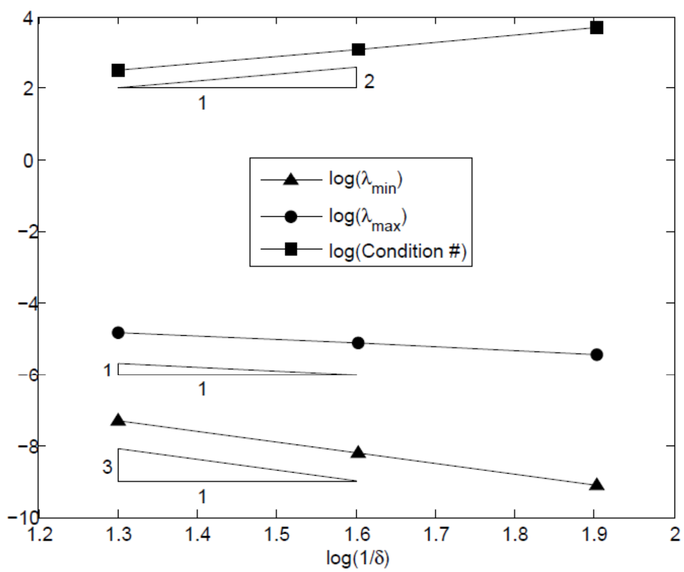
$1/h$	$1/\delta$	Piecewise Constant Shape Functions $\lambda_{\min}$	$\lambda_{\max}$	Condition #	Piecewise Linear Shape Functions $\lambda_{\min}$	$\lambda_{\max}$	Condition #
2000	20	1.94E-07	6.07E-05	3.13E+02	1.94E-07	6.07E-05	3.13E+02
4000	20	9.69E-08	3.04E-05	3.13E+02	9.69E-08	3.04E-05	3.14E+02
8000	20	4.84E-08	1.52E-05	3.14E+02	4.84E-08	1.52E-05	3.14E+02

(b) Constant h, vary δ.

$1/h$	$1/\delta$	Piecewise Constant Shape Functions $\lambda_{\min}$	$\lambda_{\max}$	Condition #	Piecewise Linear Shape Functions $\lambda_{\min}$	$\lambda_{\max}$	Condition #
8000	20	4.84E-08	1.52E-05	3.15E+02	4.84E-08	1.52E-05	3.14E+02
8000	40	6.24E-09	7.61E-06	1.22E+03	6.24E-09	7.60E-06	1.22E+03
8000	80	7.92E-10	3.80E-06	4.80E+03	7.91E-10	3.80E-06	4.80E+03



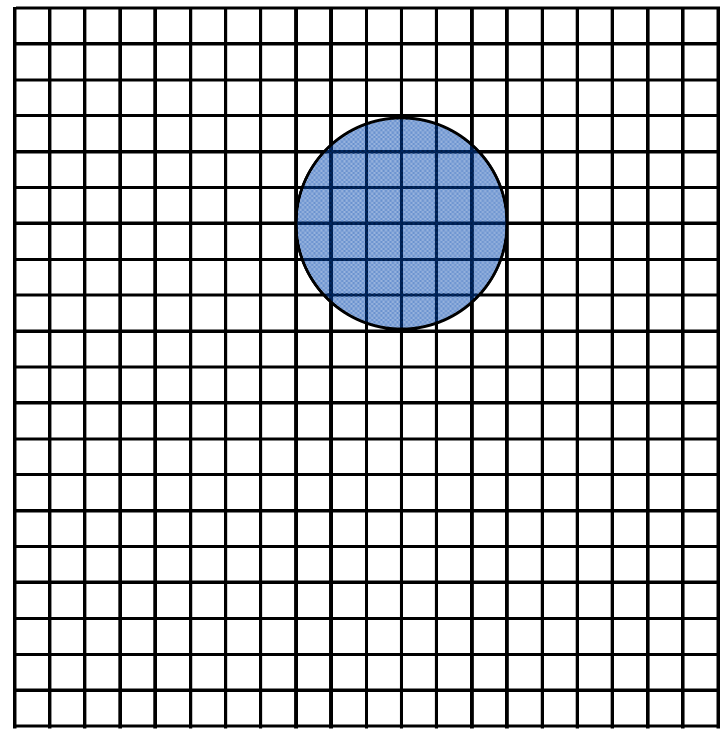
(a) Constant δ, vary h.



(b) Constant h, vary δ.

Nonlocal Weak Form – 2D

- Let $\Omega = (0,1)\times(0,1)$, $\mathcal{B}\Omega = [-\delta,0]\cup[1, \delta]$.
- $u=0$ on $\mathcal{B}\Omega$
- Let $C(x,x') = \begin{cases} 1 & \text{if } \|x - x'\| \le \delta \\ 0 & \text{otherwise} \end{cases}$
- Weak form requires quadruple quadrature
- Integrand discontinuous!
  - Gauss quadrature not accurate
  - Adaptive quadrature (expensive)
  - Break up integral into many separate integrals where integrand continuous over each subregion
- Numerical Study
  - PW constant SFs
  - Hold δ constant, vary h
  - Hold h constant, vary δ



<u>Discussion</u>:
Is there a better way to do *accurate* nonlocal quadrature?

Nonlocal Finite Elements and Conditioning – 2D

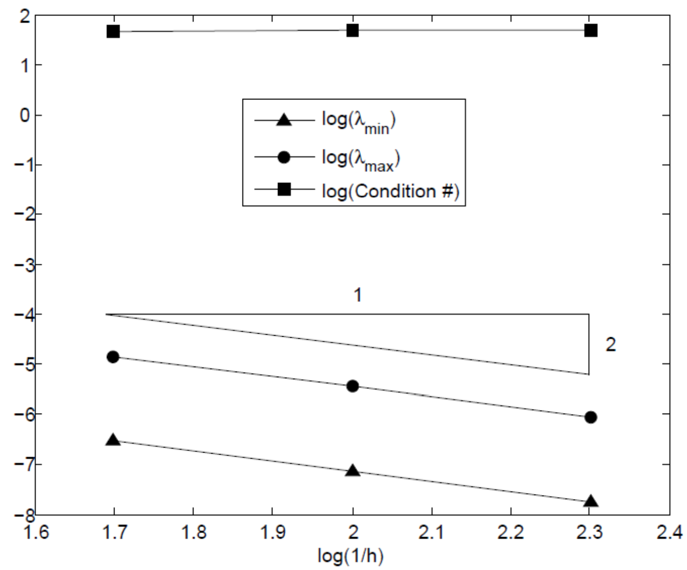
- Observations: $\kappa(K) \sim O(\delta^{-2})$, only weak h-dependence

(a) Constant δ, vary h.

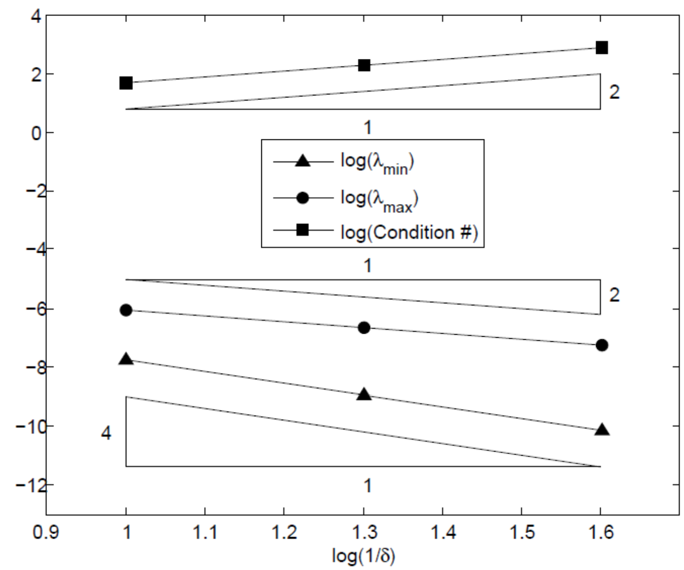
$1/h$	$1/\delta$	$\lambda_{\min}$	$\lambda_{\max}$	Condition #
50	10	2.95E-07	1.40E-05	4.77E+01
100	10	7.11E-08	3.54E-06	4.97E+01
200	10	1.75E-08	8.86E-07	5.05E+01

(b) Constant h, vary δ.

$1/h$	$1/\delta$	$\lambda_{\min}$	$\lambda_{\max}$	Condition #
200	10	1.75E-08	8.86E-07	5.05E+01
200	20	1.17E-09	2.22E-07	1.90E+02
200	40	7.63E-11	5.50E-08	7.21E+02



(a) Constant δ, vary h.



(b) Constant h, vary δ.

Summary

- ❑ **Mercifully brief review of peridynamics**
- ❑ **Applications**
  - ❑ Fracture, fragmentation, failure
- ❑ **Codes**
  - ❑ EMU, PDLAMMPS, Peridigm, more
- ❑ **Discretizations & Numerical Methods**
  - ❑ Particle-like discretization of strong form
- ❑ **Peridynamic Finite Elements**
  - ❑ Peridynamic weak forms
  - ❑ Conditioning results
- ❑ **Peridynamic Domain Decomposition**
  - ❑ Peridynamic Schur Complement
  - ❑ Conditioning results

- ❑ **Thank you!**

- ❑ **Questions for me…?**

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