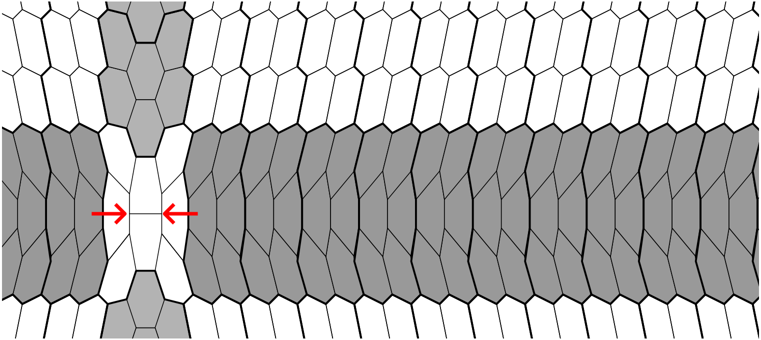
Non-vertex-to-vertex tilings by convex hexagons

Question: Is there a tiling by convex hexagons of unit area and bounded perimeter which is not vertex-to-vertex? (Nothing is required about congruence or incongruence)

Answer: Yes.



This tiling is made from copies of five different hexagons.

But in this example there are only two points where the tiling is not vertex-to-vertex (red arrows). Apart from that all tiles are vertex-to-vertex.

Problem: Find a tiling of the Euclidean plane $\mathbb{R}^2$ by convex hexagons of unit area and bounded perimeter such that there are more than two non-vertex-to-vertex situations.

Can there be infinitely many non-vertex-to-vertex situations?