There are tilings of the Euclidean plane by convex heptagons:

...but they get pretty long and thin.

I.e., the tiling is not normal.
Question: Is there a tiling by convex hexagons of unit area and bounded perimeter which is not vertex-to-vertex? (Nothing is required about congruence or incongruence)
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Answer: Yes.
This tiling is made from copies of five different hexagons.

But in this example there are only two points where the tiling is not vertex-to-vertex (red arrows). Apart from that all tiles are vertex-to-vertex.

**Problem:** 1. Find a tiling of the Euclidean plane $\mathbb{R}^2$ by convex hexagons of unit area and bounded perimeter such that there are more than two non-vertex-to-vertex situations.

2. Can there be infinitely many non-vertex-to-vertex situations?
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2. Can there be infinitely many non-vertex-to-vertex situations?
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Answers:

1.: Yes, there are for any $n \geq 2$

2.: Not possible
Observation: Non-vtv situations correspond to 7-gons (8-gons,...)

Theorem (Stehling 1989, Akopyan 2018)
If the plane is tiled with convex polygons, which have at least six sides, have area at least $A$ and diameter not greater than $D$, then the sum of "additional vertices" of the tiles (= non-vtv situations) is bounded by $2\pi D^2 A - 6$.

Normal tiling, hence $A$ bounded from below, $D$ bounded from above, yields: number of non-vtv situations bounded.
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If the plane is tiled with convex polygons, which have at least six sides, have area at least $A$ and diameter not greater than $D$, then the sum of "additional vertices" of the tiles (= non-vtv situations) is bounded by $2\pi D^2 / A - 6$.

Normal tiling, hence $A$ bounded from below, $D$ bounded from above, yields: number of non-vtv situations bounded.