Hofstetter Kurt's ~~Inductive Rotation Tilings~~ Hofstetter Tilings

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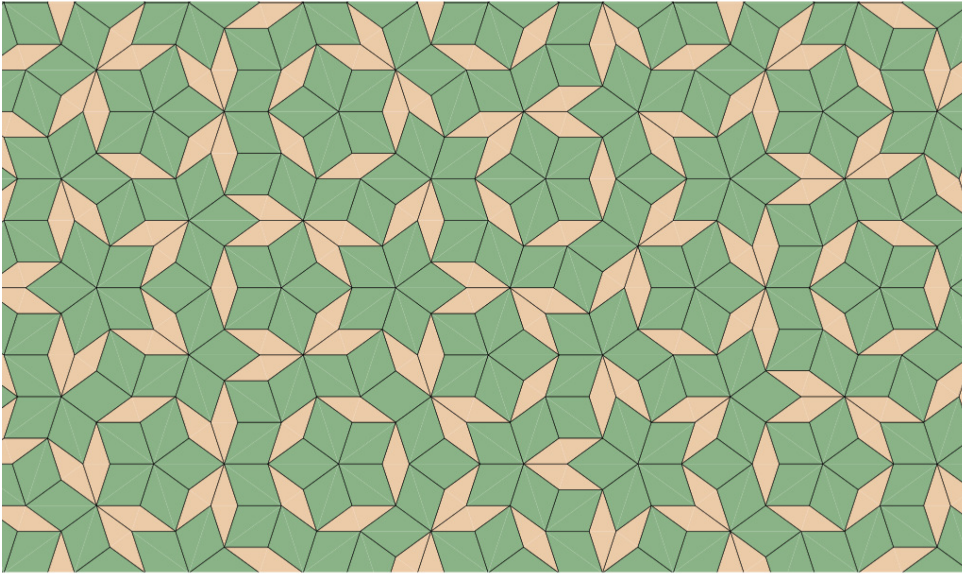
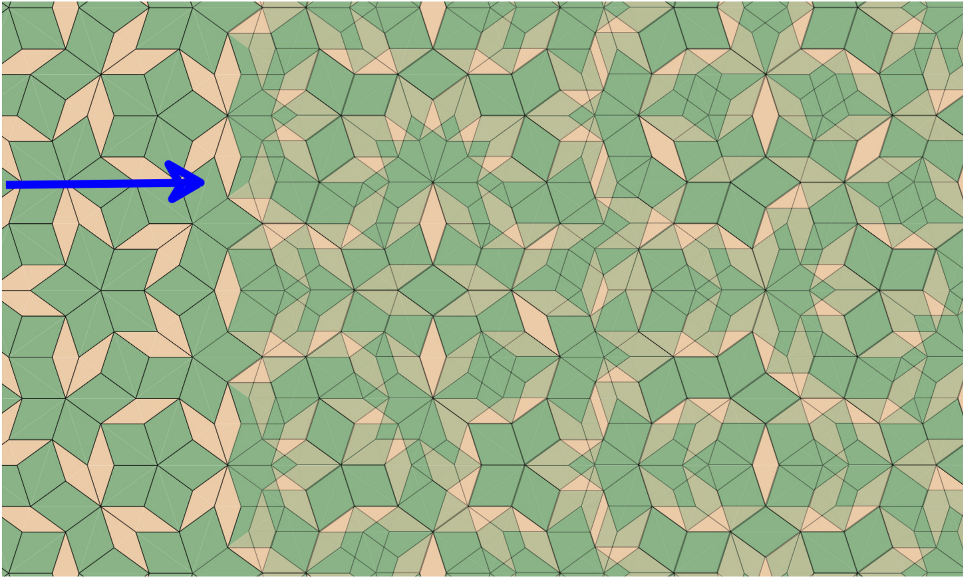
Geometrietag Jena 5. Dec. 2015

1. Aperiodic tilings

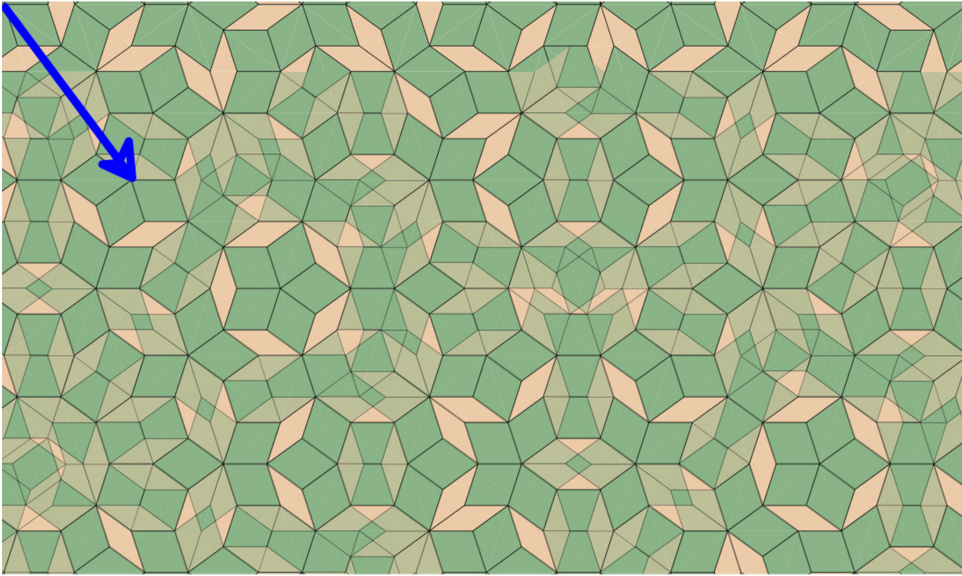
Aperiodic Tiling



Aperiodic

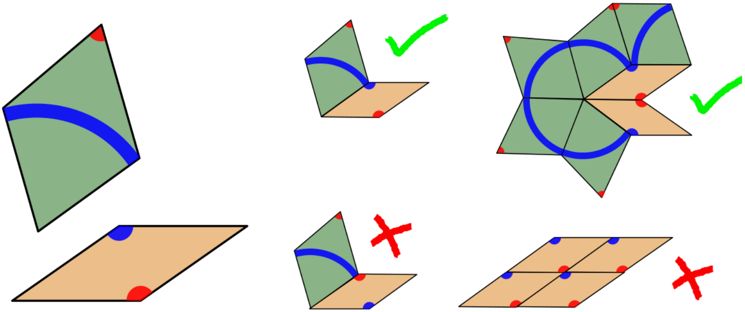


Aperiodic



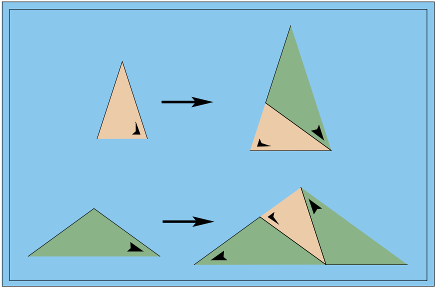
Local rules

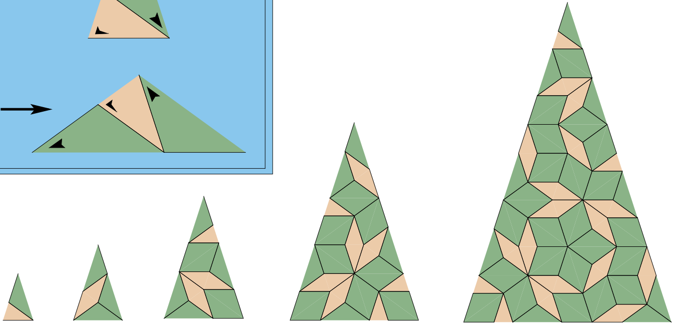
Penrose tiling: can be generated by local rules
(forces aperiodicity)



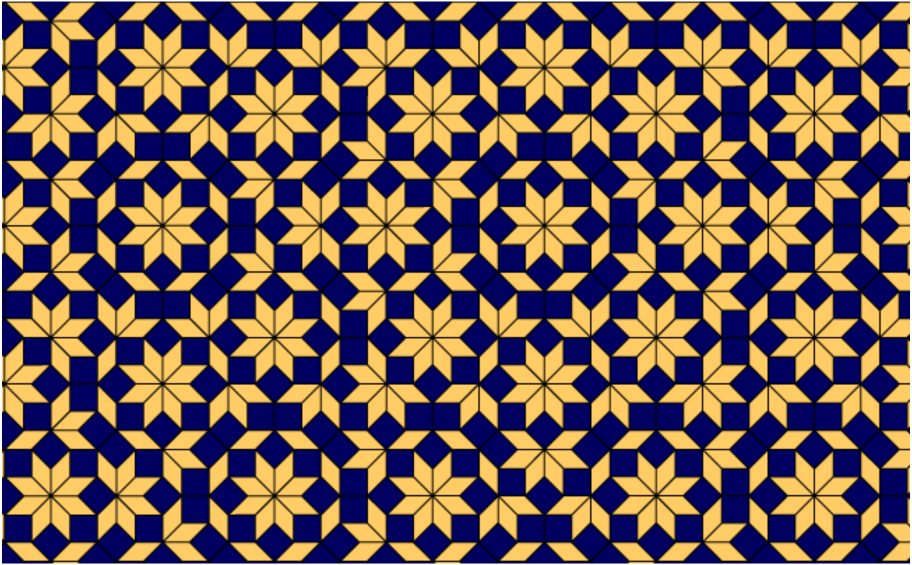
Substitution tilings:

Penrose tiling: can also be generated by a substitution rule

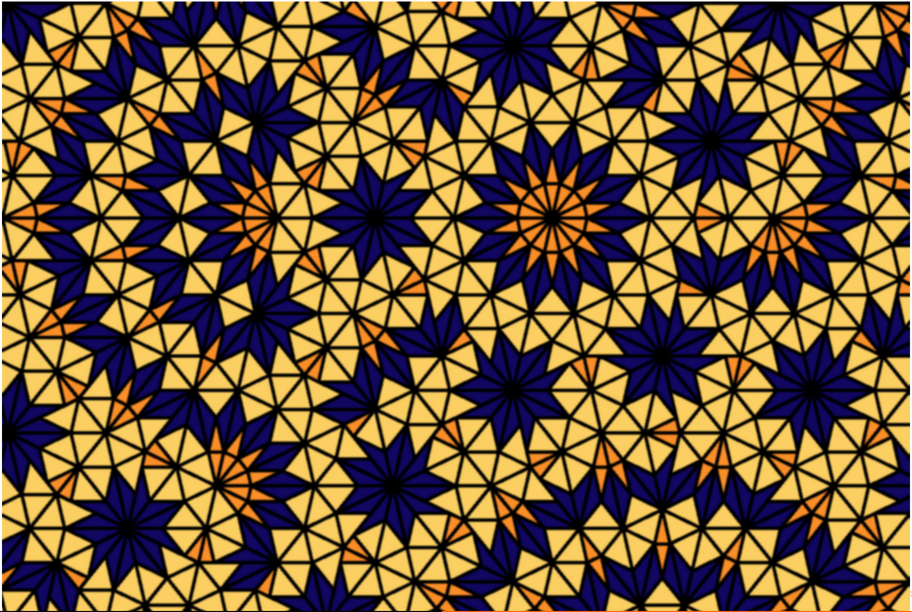




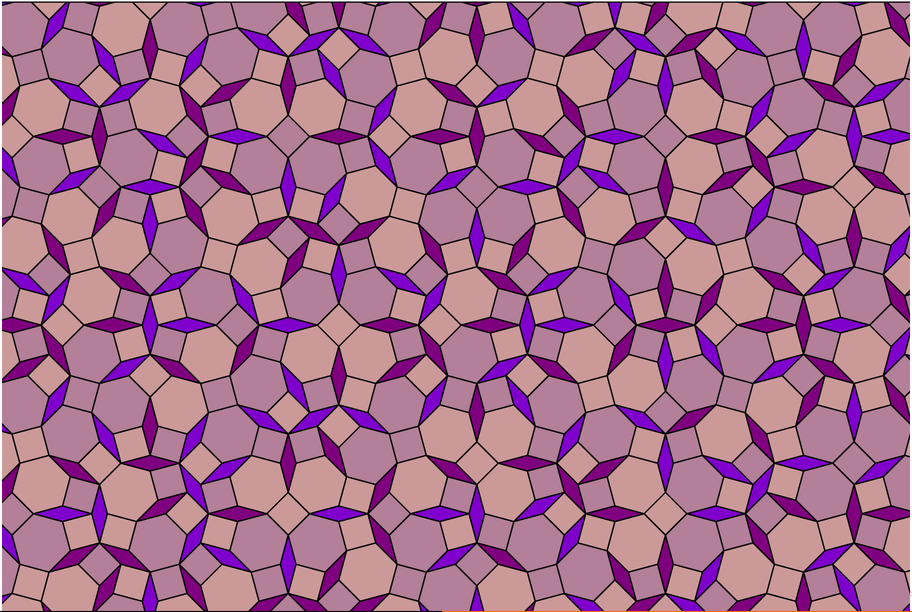
Other examples: Ammann-Beenker tiling



Other examples: Buffalo tiling



Other examples: Socolar's 12-fold tiling

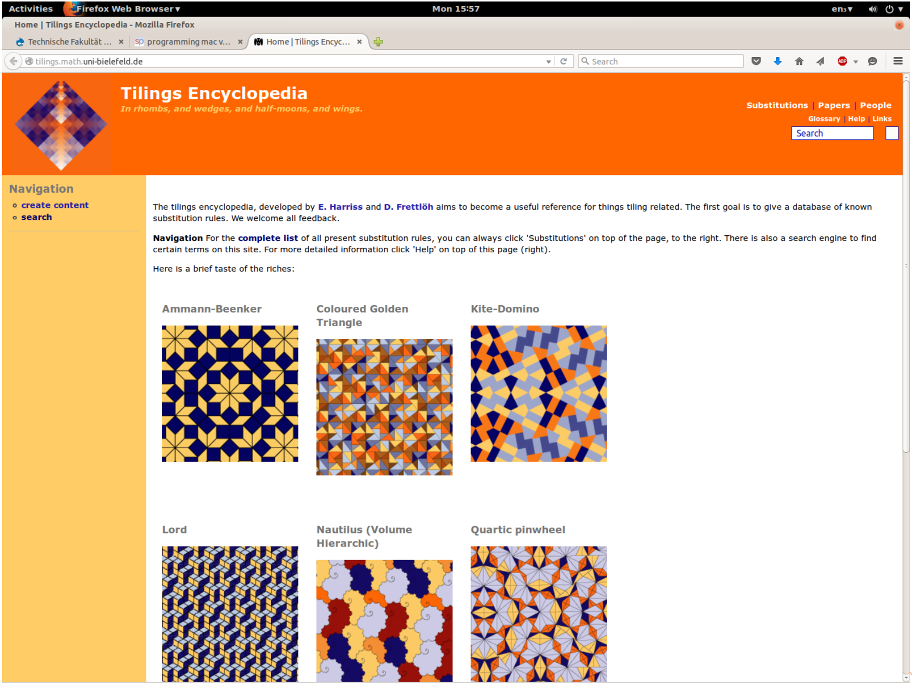


Aperiodic tilings with a high degree of local and global order can be generated by

- Local matching rules
- Substitution rules
- (Cut and project method, more technical)

Penrose: 1972, others in the 70s and 80s.

My contribution (with Edmund Harriss and Franz Gähler):



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Tilings Encyclopedia

In rhombs, and wedges, and half-moons, and wings.

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Navigation

- **create content**
- **search**

The tilings encyclopedia, developed by **E. Harriss** and **D. Frettlöh** aims to become a useful reference for things tiling related. The first goal is to give a database of known substitution rules. We welcome all feedback.

Navigation For the **complete list** of all present substitution rules, you can always click 'Substitutions' on top of the page, to the right. There is also a search engine to find certain terms on this site. For more detailed information click 'Help' on top of this page (right).

Here is a brief taste of the riches:

Ammann-Beenker | **Coloured Golden Triangle** | **Kite-Domino**

Lord | **Nautilus (Volume Hierarchic)** | **Quartic pinwheel**

2. ~~Inductive rotation~~ Hofstetter Tilings

In 2010 I received an email:

Terminal

ALPINE 2.02(1266) MESSAGE TEXT ...ehl-zwei2010 Message 264 of 703 65% ANS

Sehr geehrter Herr Prof. Frettlöh,

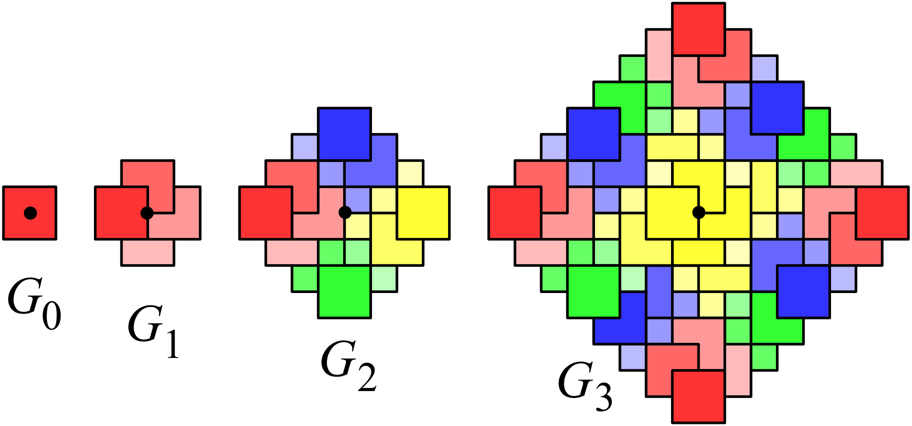
ich beschäftige mich in meiner künstlerischen Arbeit intensiv mit
Parallelität und Kreislauf und habe folgendes System der Induktiven Rotation
(siehe anbei) entdeckt um "lebendige" Muster / Parkettierungen / tilings zu
kreieren. Dieser Entdeckung sind die wissenschaftlichen Veröffentlichungen
meiner geometrischen Konstruktionen im mathematischen Journal
ForumGeometricorum (links siehe anbei) vorangegangen, die mich mit
Sicherheit stimuliert haben derartige Muster zu entwickeln.

Da ich via Internet herausgefunden habe, dass Sie sich wissenschaftlich mit
Parkettierungen aller Art beschäftigen und daran auch enzyklopädisch
arbeiten, wollte ich Ihnen o.a. Methode mit der Bitte um Ihre Stellungnahme
bzw. Einordnung, zur Kenntnis bringen.

Mit den besten Grüßen aus Wien,
Hofstetter Kurt, e.h.
http://www.sunpendulum.at/hofstetterkurt.html [www.sunpendulum.at]

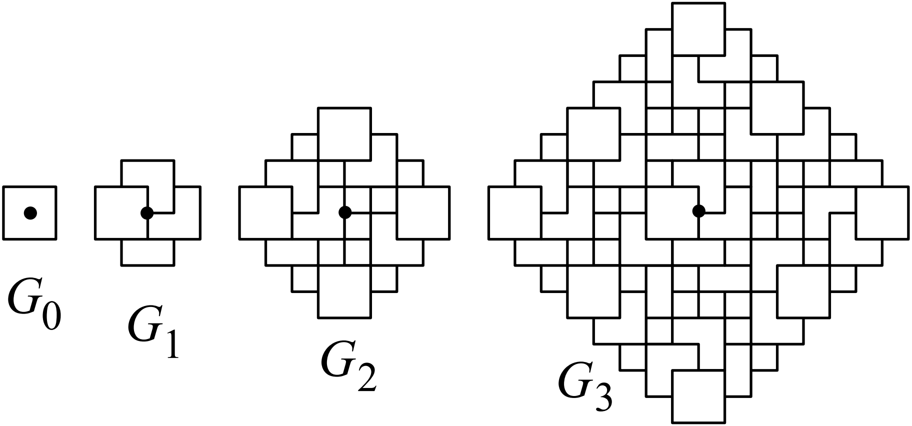
- scientific papers @ Forum Geometricorum ISSN 1534-1178:
2008, **A simple compass-only construction of the regular pentagon**
[forumgeom.fau.edu]

Inductive rotation



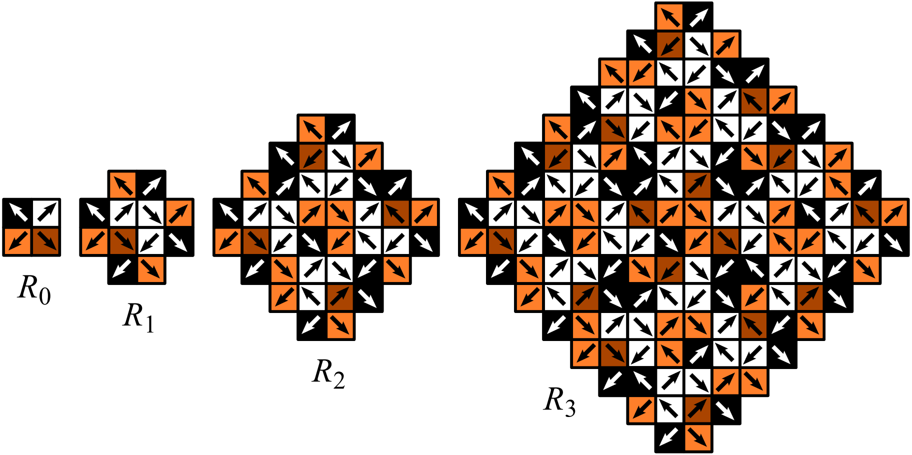
Repeating the iteration fills larger and larger regions.

Inductive rotation

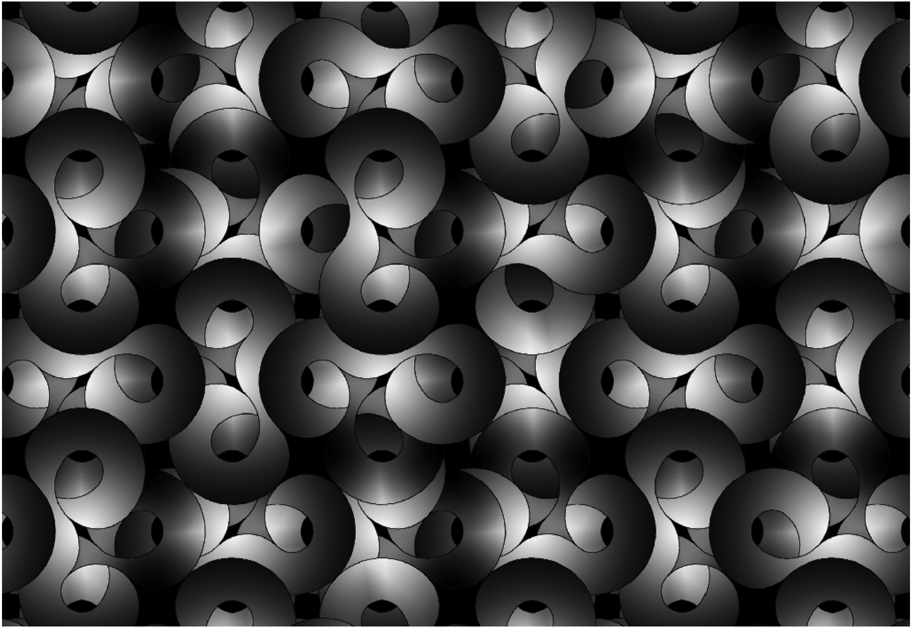


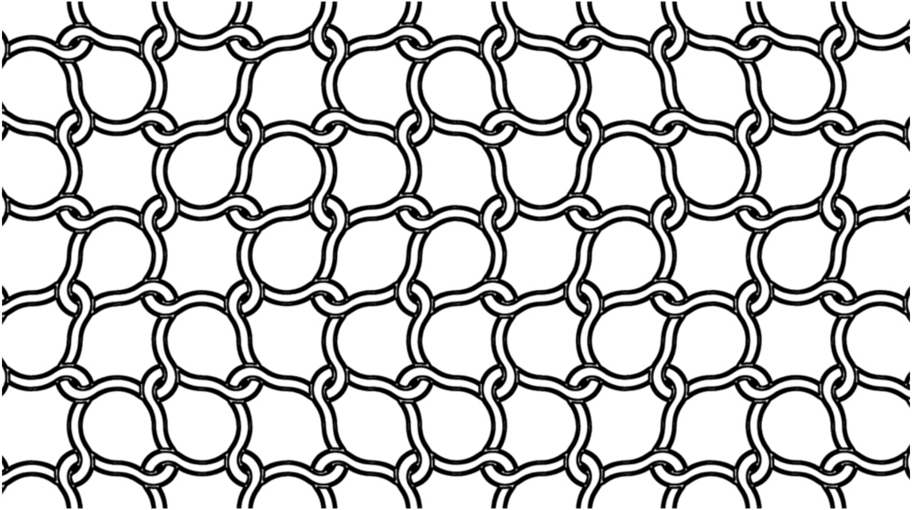
A new construction for aperiodic tilings!?

Decorations of the underlying yields variants of the “naked” tiling (last slide): e.g. the “arrowed” tiling.



...and others: ⧅, ⊡, ⊟... including periodic tilings: ⊞.





Due to some constraints we really got things started in 2014.

Questions:

- *Are* the tilings aperiodic?
- Can they be generated by a cut-and-project method?
- How are the tilings related (e.g. naked vs arrowed)
- ...

We were able to prove answers to 1 and 2 for the arrowed version.
Answers: yes and yes.

Central results:

Theorem (1)

The arrowed tilings are limitperiodic.

Theorem (2)

The arrowed tilings can be generated by a substitution rule.

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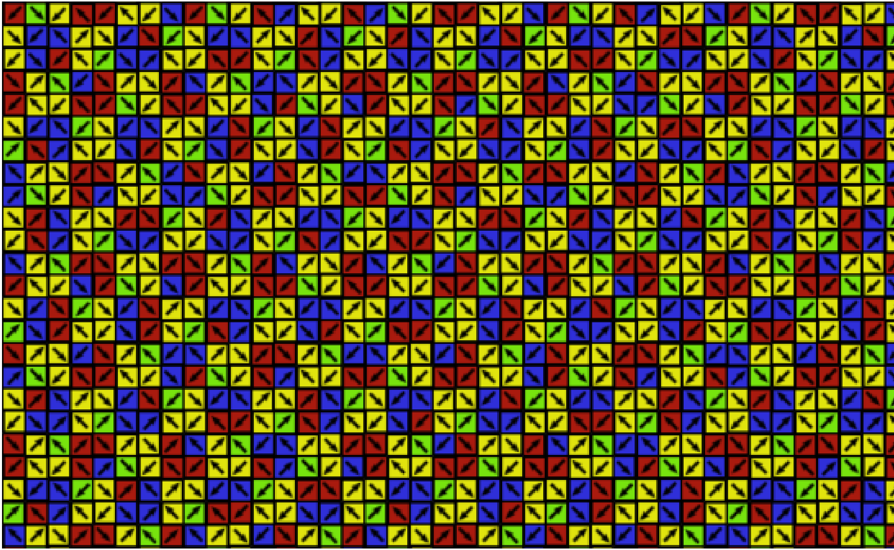
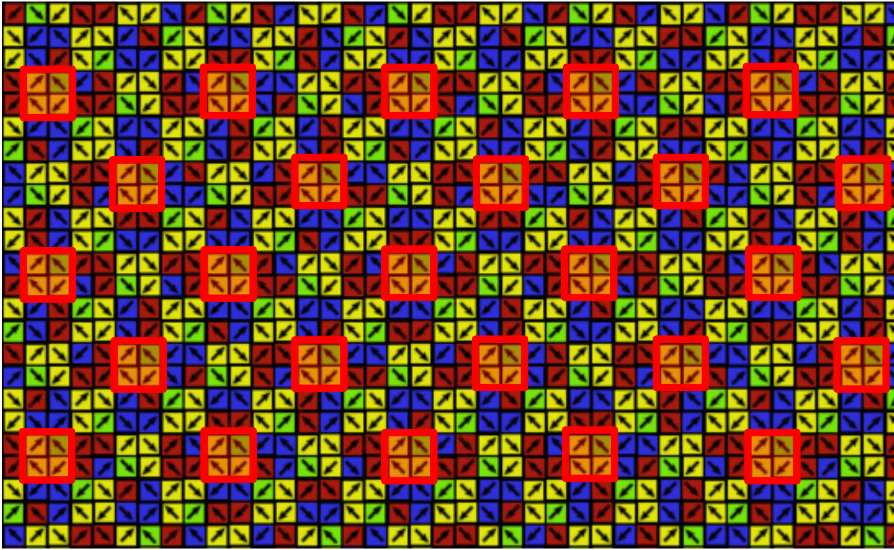
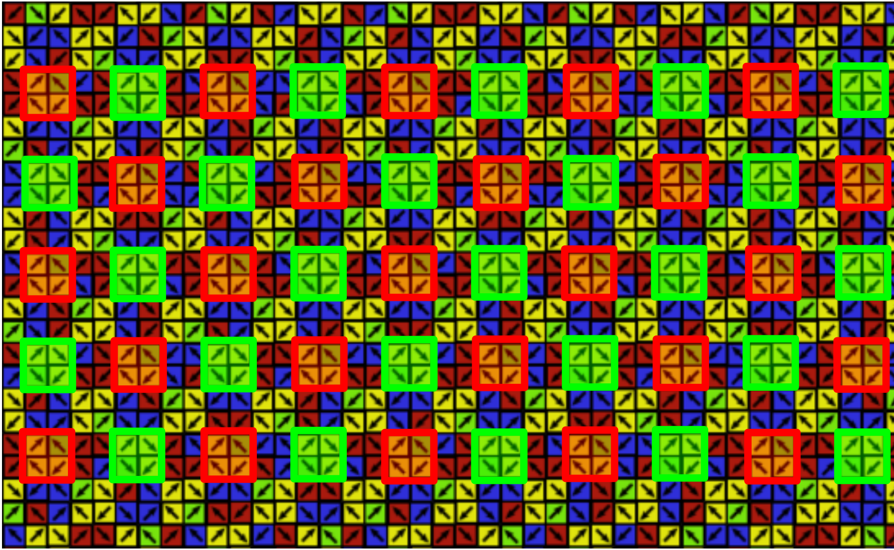
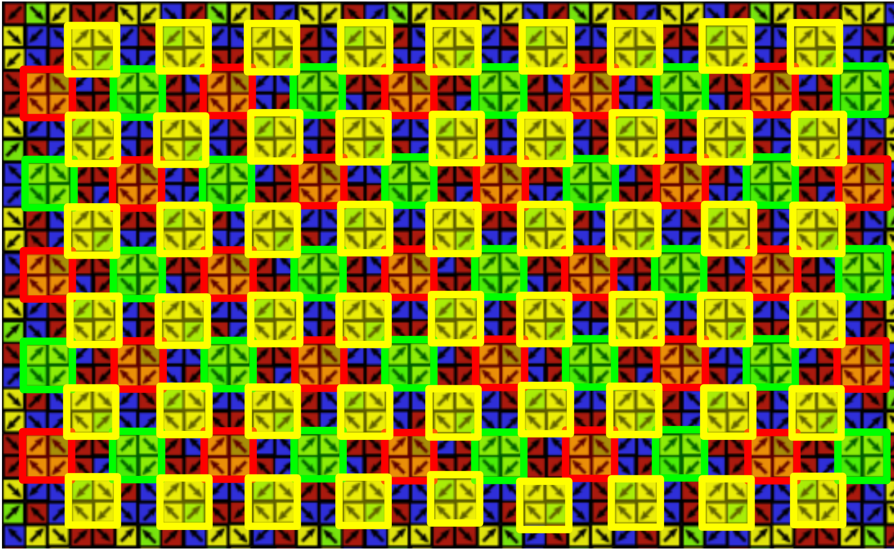
Remark: Theorem 2 is a pity, since this *new* method generates tilings that can also be obtained by the *old* method.

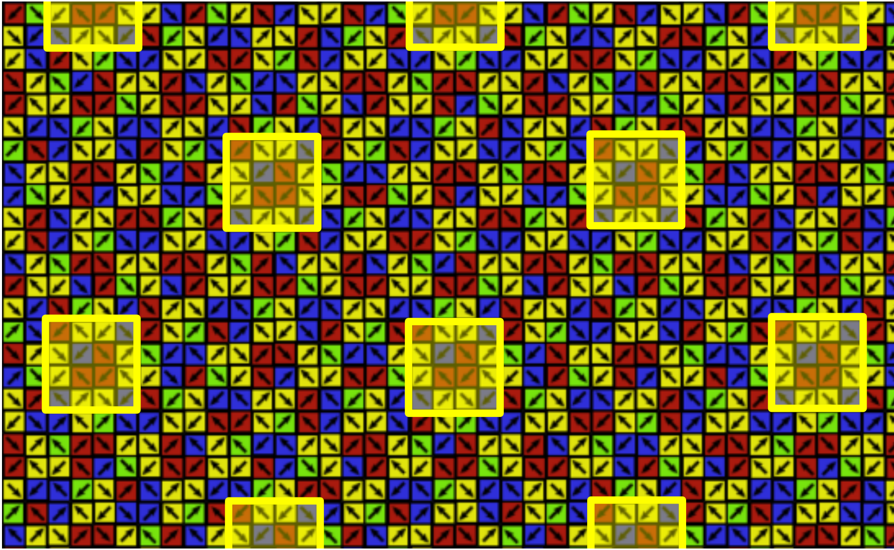
Anyway. What does “limitperiodic” mean?











...and so on. The entire tiling is the union of periodic sub-tilings. This property is called “limitperiodic”. (In dynamical systems aka ”Toeplitz structures”)

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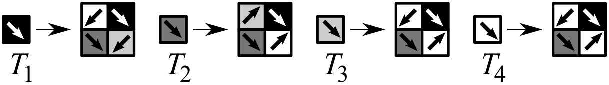
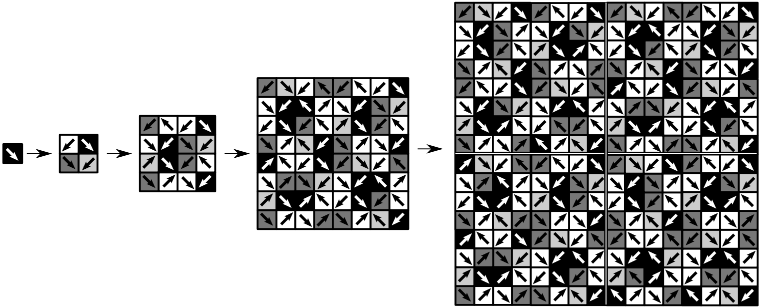
Once Theorem 1 is proven, this allows to prove Theorem 2.

The substitution rule:



...generates the same (infinite) tilings as Kurt’s inductive rotation method.



Once Theorem 2 is proven, the well-developed machinery for substitution tilings can be applied. Yields e.g.

Theorem (3)

The arrowed tiling is aperiodic.

Theorem (4)

The arrowed tiling can be generated by a cut-and-project method.

Theorem 3 can be proven by a “classical” result (1984) which says essentially:

If the substitution rule has a unique inverse (by local means) then the tiling is aperiodic.

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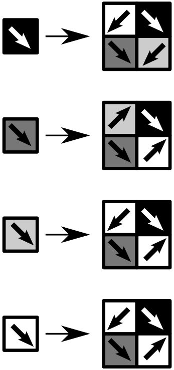
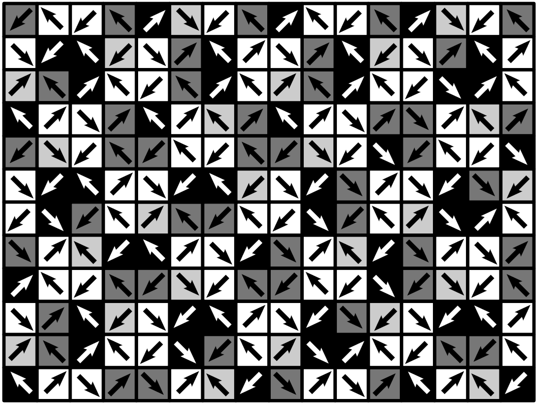
If the substitution rule has a unique inverse (by local means) then the tiling is aperiodic.

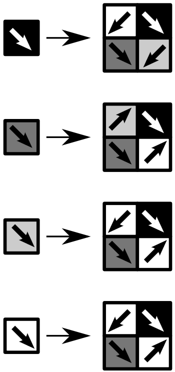
In even plainer words:

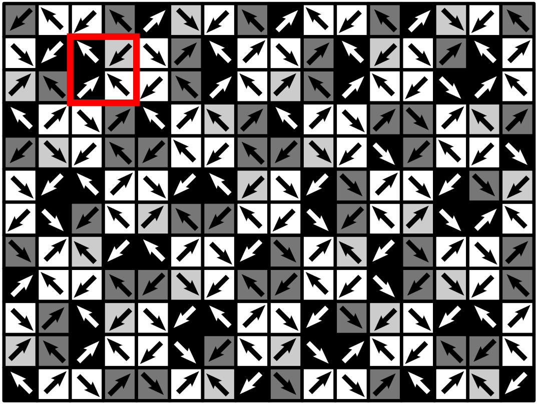
If one can identify the “previous generation” of the substitution tiling in a unique way then the tiling is aperiodic.

Here:









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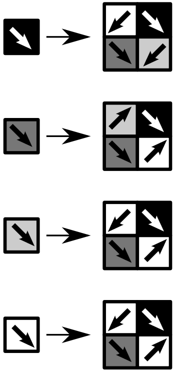

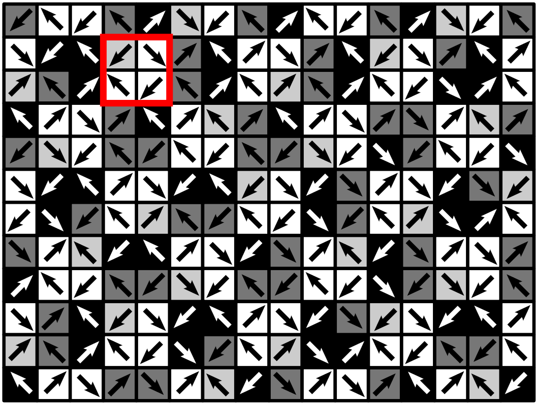

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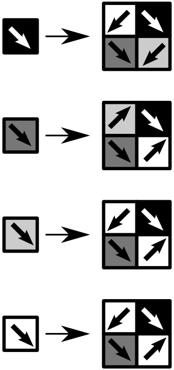

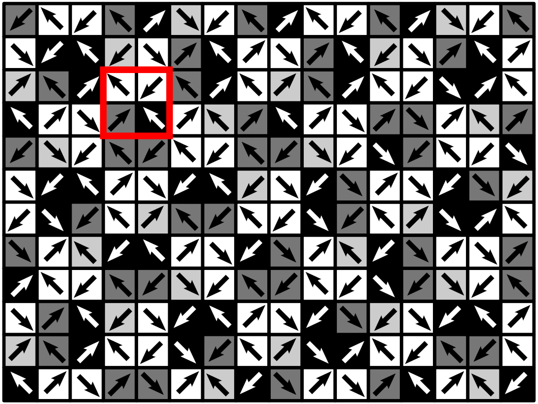

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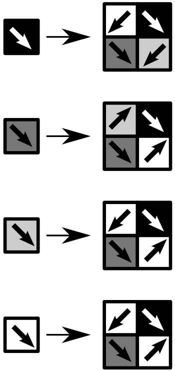
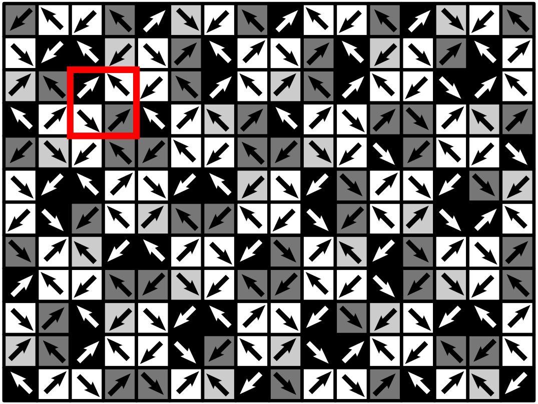
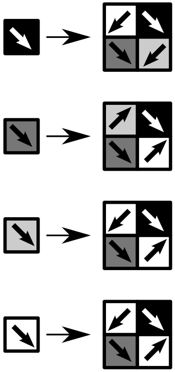
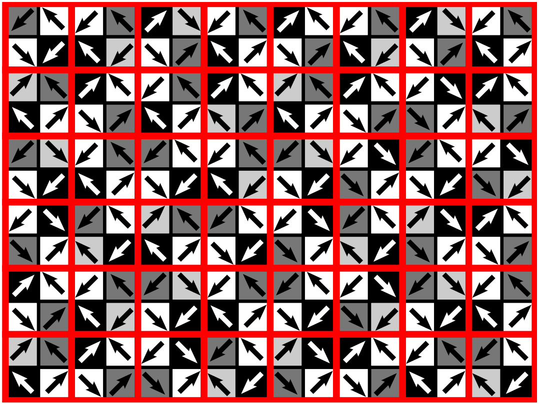




Yes.





Bingo!

In a similar manner we can use general results to compute e.g. the relative frequencies of the tiles in the tiling:

The relative frequencies of the tiles are the entries of the normalised eigenvector of the dominant eigenvalue of the substitution matrix

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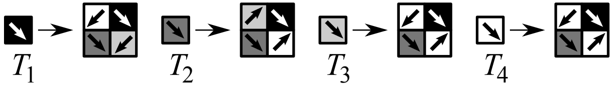

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 2 & 2 \end{pmatrix}$$

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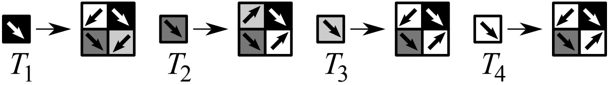

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 2 & 2 \end{pmatrix}$$

Relative frequencies: **2:2:1:3**

Theorem 4 is obtained in a similar manner, using a general result applied to this situation (Lee-Moody-Solomyak 2003):

If [some technical condition is fulfilled] then the tilings are cut-and-project tilings.

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More in

D. Frettlöh, K. Hofstetter:
Inductive rotation tilings, *Proc. Steklov Inst.* 288 (2015) 269-280;
`arXiv`:1410.0592

The article mentioned answers several questions on the *arrowed* tiling.

Several further questions remain open. E.g.

- What about other tile decorations, e.g. the naked version? (probably everything the same)
- What about variants with 3-fold rotations and 6-fold rotations, rather than 4-fold?
  (3-fold: probably analoguously, 6-fold: no idea)

Thank you!