Dr. D. Frettlöh 16.10.2018

Formal Logic — Exercise Sheet 1

Exercise 1: (Elves and Vampires)
In a distant world there are only two types of inhabitants: Elves, who always tell the truth, and vampires, who always lie. All problems (a)-(e) of this Exercise take place in this world.

(a) In this problem there are two people, A and B, each of whom is either an elve or a vampire. A makes the statement: "At least one of us is a vampire." What are A and B?

(b) Suppose A says, "I am a vampire, but B isn't." What are A and B?

(c) Now we have three people, A, B, C, each of whom is either an elve or a vampire. A and B make the following statements:

A: All of us are vampires. B: Exactly one of us is an elve.

What are A, B, C?

(d) Again we have three people A, B and C, each of whom is a elve or a vampire. Two people are said to be of the same type if they are both elves or both vampires. A and B make the following statements:

A: B is a vampire. B: A and C are of the same type.

What is C?

(e) Again three people A, B, and C. A says "B and C are of the same type." Someone then asks C, "Are A and B of the same type?" What does C answer?

Exercise 2: (Borromean formulas)
(a) Find three formulas F_1, F_2, F_3 such that $F_i \wedge F_j$ is satisfiable for all choices of $1 \le i < j \le 3$, but $F_1 \wedge F_2 \wedge F_3$ is not satisfiable.

(b) Find n formulas $F_1, F_2, \ldots, F_n$ for n odd, such that $F_{i_1} \wedge F_{i_2} \wedge \cdots \wedge F_{i_{n-1}}$ is satisfiable for all choices of $\{i_1, i_2, \ldots, i_{n-1}\} \subseteq \{1, 2, \ldots, n\}$, but $F_1 \wedge F_2 \wedge \cdots \wedge F_n$ is not satisfiable.

Bonus points: solve (b) for n even.

Exercise 3: (XOR and $\Leftrightarrow$)
(a) Show that the "exclusive or" $A \oplus B$ (where $\mathcal{A}(A \oplus B) = 1$ if and only if $\mathcal{A}(A) \neq \mathcal{A}(B)$) is equivalent to $(A \wedge \neg B) \vee (\neg A \wedge B)$.

(b) Show that the equivalence $A \Leftrightarrow B$ (where $\mathcal{A}(A \Leftrightarrow B) = 1$ if and only if $\mathcal{A}((A) = \mathcal{A}(B))$ is equivalent to $(\neg A \vee B) \wedge (A \vee \neg B)$.

Exercise 4: (Truth tables)
Construct truth tables for the following formulas. Are F_1 and F_2 equivalent? Are F_3 and F_4 equivalent? Are F_2 and F_3 equivalent?

$$F_1 = \neg(A \Rightarrow B), \quad F_2 = \neg\big(\neg A \vee \neg(\neg B \vee \neg A)\big), \quad F_3 = (C \wedge \neg(A \Rightarrow B)) \vee \neg(C \vee \neg A \vee B), \quad F_4 = A \Leftrightarrow (B \Leftrightarrow C)$$

Hand in your solutions until 23.10.2018 at 14:00 in post box 2183 in V3,
or via email to the tutor.

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