Dr. D. Frettlöh 30.10.2018

Formal Logic — Exercise Sheet 3

Exercise 9: (Horn formula algorithm)
(a) Apply the Marking Algorithm for Horn formulas to the following two formulas F and G. Is F (resp. G) satisfiable? If yes, please give a valuation $\mathcal{A}$ with $\mathcal{A} \models F$ (resp. $\mathcal{A} \models G$).

$$F = \neg A \wedge (\neg B \vee \neg D) \wedge (\neg C \vee \neg E \vee B) \wedge (\neg D \vee C) \wedge D \wedge (\neg C \vee \neg D \vee E)$$

$$G = (\neg A_2 \vee \neg A_4 \vee \neg A_5 \vee A_6) \wedge (\neg A_1 \vee A_2 \vee \neg A_3) \wedge \neg A_5 \wedge (\neg A_6 \vee \neg A_7 \vee A_4) \wedge A_1 \wedge (A_3 \vee \neg A_6) \wedge A_7 \wedge (\neg A_1 \vee \neg A_7 \vee A_6)$$

(b) A nonagenarian is asked how he reached this great age. His answer: "A strict diet. For each meal I obey the following rules: Whenever I eat pork I drink beer. When I eat salad I will have pork, too. To each meal I drink beer or maté, or both. If I do not eat tofu then I will have no salad. If I have beer and tofu I will not drink maté. To each meal I will have salad."

Translate his statement into a Horn formula F. (Yes, it is possible. Be creative.) Is F satisfiable? If yes, please give a valuation $\mathcal{A}$ with $\mathcal{A} \models F$.

Exercise 10: (Easy Horn formula/not a Horn formula)
(a) Show that any Horn formula (in CNF) is satisfiable if each disjunctive clause contains at least one $\neg$.

(b) Construct a formula F that has no Horn formula equivalent to F. Give a convincing reason why there is no Horn formula equivalent to your formula.

Exercise 11: (satisfiable vs tautology)
Prove or give a counterexample:

(a) If F is a tautology and $F \Rightarrow G$ is a tautology, then G is a tautology.

(b) If F is satisfiable and $F \Rightarrow G$ is satisfiable, then G is satisfiable.

(c) If F is satisfiable and $F \Rightarrow G$ is a tautology, then G is satisfiable.

(d) If F is satisfiable and $F \Rightarrow G$ is a tautology, then G is a tautology.

Exercise 12: (Infinitely many formulas)
Find all valuations for $A_1, A_2, \ldots$ satisfying the infinite set of formulas

$$\{A_1 \vee A_2, \neg A_2 \vee \neg A_3, A_3 \vee A_4, \neg A_4 \vee \neg A_5, A_5 \vee A_6, \ldots\}$$

(Hint: there are more than seven.)

Hand in your solutions until 6.11.2018 at 14:00 in post box 2183 in V3,
or via email to the tutor.

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