

Formal Logic — Exercise Sheet 5**Exercise 17: (MMORG)**

In the onlinegame WARSTARS™ there are several goods that can be traded. It is known that you can always trade wood and dilithium for strontium and gold (you need to give both wood and dilithium, and will in turn receive both strontium and gold). You can also trade neutrinos and gold for redstone (again you need to give both neutrinos and gold, and will receive redstone), or strontium and heavy water for neutrinos. You have an unlimited amount of wood, dilithium and heavy water. Use the tableau calculus to show that you can obtain redstone: translate the information above into a formula F and apply the tableau calculus in order to show that “you can obtain redstone” is a consequence of F .

Please do not simplify F before applying the tableau calculus. I know it is much simpler, but this one should serve as an example for the tableau calculus!

Exercise 18: (Efficiency of Resolution)

(a) Let F be a formula in CNF using n different atomic formulas. Show that in the resolution algorithm the number of resolution steps is at most $O(n^2)$ if all clauses in the CNF of F have length at most two.

(This shows essentially that the runtime for 2SAT is $O(n^2)$, i.e., 2SAT is efficiently decidable.)

(b) Let F be a clause set with m clauses containing the atomic formulas A_1, \dots, A_n . what is the maximal value of $|Res^*(F)|$? (Here $|M|$ denotes the number of elements of a set M .)

(c) Prove that there is $k \in \mathbb{N}$ such that

$$Res^k(F) = Res^{k+1}(F) = \dots = Res^*(F).$$

Exercise 19: (Relations)

Which of the following relations are equivalence relations (on the corresponding sets)? Give a convincing reason why they are, or provide a counterexample. Let B denote the set of all people living in Bielefeld. For the ones that are equivalence relations: list all equivalence classes.

1. $R = \{(a, b) \mid a, b \in B, a \text{ and } b \text{ live in the same street.}\}$ on B .
2. $R = \emptyset$ on B .
3. $R = B \times B$ on B .
4. $R = \{(a, b) \mid a, b \in \mathbb{Z}, a \leq b\}$ on \mathbb{Z} .
5. $R = \{(a, b) \mid a, b \in \mathbb{Z}, |a - b| \leq 2\}$ on \mathbb{Z} .
6. $R = \{(a, b) \mid a, b \in \mathbb{Z}, \exists k \in \mathbb{Z} : |a - b| = 7k\}$ on \mathbb{Z} .

Exercise 20: (Dr Who pushing buttons)

Dr Who encounters some device about to explode, thus destroying all life, time and space existing. It has three buttons. The device can be shut down (hence not explode) by pressing the correct button. It is known that one should press at most one of the first two buttons (otherwise the device explodes, with the consequences mentioned above). If the third button is pressed then one needs to press the second button, too. If one pushes the second button but not the first one, one should not press the third button. In order to achieve anything one should at least press one button.

Translate the situation into a formula F . Use the tableau method to determine which button should certainly not be pressed at all. (Note: It is not possible to determine the correct button)