Dr. D. Frettlöh 20.11.2018

Formal Logic — Exercise Sheet 6

Exercise 21: (Models?)
Which of the following structures are models for

$$F = \exists x\, \exists y\, \exists z\, \big(P(x,y) \wedge P(z,y) \wedge P(x,z) \wedge \neg P(z,x)\big),$$

which are not? Please give a convincing reason for each of your answers.

(a) $U_{\mathcal{A}} = \mathbb{N}$, $P^{\mathcal{A}} = \{(m,n) \mid m,n \in \mathbb{N},\ m < n\}$

(b) $U_{\mathcal{A}} = \mathbb{N}$, $P^{\mathcal{A}} = \{(n,n+1) \mid n \in \mathbb{N}\}$

(c) $U_{\mathcal{A}} = \{f : \mathbb{R} \to \mathbb{R}\}$ (all functions from $\mathbb{R}$ to $\mathbb{R}$), $P^{\mathcal{A}} = \{(f,g) \mid f = g', g \text{ is differentiable}\}$

(d) $U_{\mathcal{A}} =$Pot($\mathbb{N}$), (the set of all subsets of $\mathbb{N}$), $P^{\mathcal{A}} = \{(A,B) \mid A,B \subset \mathbb{N},\ A \subseteq B\}$

Exercise 22: (Models and non-models)
List all partial formulas of the following formula, and all terms, and write down the matrix of F.

$$F = \forall x\, \exists y\, P(x,y,f(x))$$

Find two structures that are not models for F, and two structures that are models for F.

Exercise 23: (Reflexive, symmetric, transitive)
Consider the formulas

$$F_1 = \forall x\, P(x,x), \quad F_2 = \forall x\, \forall y\, \big(P(x,y) \Rightarrow P(y,x)\big), \quad F_3 = \forall x\, \forall y\, \forall z\, \big((P(x,y) \wedge P(y,z)) \Rightarrow P(x,z)\big).$$

Show that none of these formulas is a consequence of the other two by constructing structures where F_1 and F_2 are true, but F_3 is not; respectively where F_1 and F_3 are true, but F_2 is not; respectively where F_2 and F_3 are true, but F_1 is not.

Exercise 24: (Small universes)
(a) Find a formula F of first-order logic without free variables such that F is satisfiable only if $U_{\mathcal{A}}$ has at least three elements. (I.e., F is unsatisfiable for all $U_{\mathcal{A}}$ where $U_{\mathcal{A}}$ has only one or two elements.)

(b) Find a formula F of first-order logic with identity, and without free variables, such that for all $\mathcal{A}$ with $\mathcal{A} \models F$ holds that $U_{\mathcal{A}}$ has at most two elements.

Hand in your solutions until 27.11.2018 at 14:00 in post box 2183 in V3,
or via email to the tutor.

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