Formal Logic   —   Exercise Sheet 7

Exercise 25: (Not a law)
Show that the two following pairs of formulas are not equivalent to each other, respectively, by providing a counterexample (i.e., a structure such that the formula is false) for each one.

(a) $(\forall x\ F) \vee (\forall x\ G) \quad \not\equiv \quad \forall x(F \vee G)$,

(b) $(\exists x\ F) \wedge (\exists x\ G) \quad \not\equiv \quad \exists x(F \wedge G)$.

Exercise 26: (Equivalence vs consequence)
(a) Show that $(\exists x\ P(x)) \Rightarrow P(y) \equiv \forall x\ (P(x) \Rightarrow P(y))$.

(b) Show that $F = \forall x\ \exists y\ P(x,y)$ is not a consequence of $G = \exists x'\ \forall y'\ P(x',y')$, and that G is not a consequence of F.

(c) Show that $\forall x\ \exists y\ P(x,y)$ is a consequence of $\exists x'\ \forall y'\ P(y',x')$, but not vice versa.

Exercise 27: (Skolem normal form)
Transform the following formulas in Skolem normal form such that the matrix of the formula has CNF.

$$F = \big(\neg \forall w\ P(a,w)\big) \vee \forall x\ \exists y\ \forall z\ Q(f(x),y)$$

$$G = \Big(\neg \exists x\ \big(P(x,z) \vee \forall y\ Q(x,f(y)))\big)\Big) \vee \forall y\ P(g(x,y),z)$$

$$H = \forall z\ \exists y\ \big(P(x,g(y),z) \vee \neg \forall x\ Q(x)\big) \wedge \neg \forall z\ \exists x\ \neg R(f(x,z),z)$$

Exercise 28: (Unexpected hanging paradox)
A judge tells a condemned prisoner that he will be hanged at noon on one weekday in the following week but that the execution will be a surprise to the prisoner. He will not know the day of the hanging until the executioner knocks on his cell door at noon that day.

Translate this situation into a formula F in first-order logic with identity. Let G be the statement that the prisoner cannot be hanged at any day. Transform the formula $F \wedge \neg G$ into Skolem normal form, along the lines of Algorithm 2.1.

If you want to try yourself, go ahead. If you are in doubt, hints here:

`https://www.math.uni-bielefeld.de/~frettloe/teach/logik/spoiler-a28.html`

Later we will try to solve this paradox.

Hand in your solutions until 4.12.2018 at 14:00 in post box 2183 in V3,
or via email to the tutor.

Tutor: Oliver Tautz   otautz@techfak.uni-bielefeld.de