Dr. D. Frettlöh 4.12.2018

Formal Logic — Exercise Sheet 8

Exercise 29: (Drinker paradox)
In each (nonempty) pub there is someone such that, if this person is drinking, then everyone in the pub is drinking.

Translate the statement above into a formula in first-order logic and prove that it is valid using the resolution calculus. *(Hence this is not really a paradox, it sounds only like one.)*

Exercise 30: (Barber paradox)
In a remote small town there are two strict rules for barbers:

1. Each barber shaves all those who do not shave themselves.

2. No barber shaves someone who shaves himself.

Show that there is no barber in this town, by translating 1. and 2. and "there is a barber" into formulas F, G and H in first-order logic and use the resolution calculus in order to show that $F \wedge G \wedge H$ is unsatisfiable. *(Thus this one (1 and 2 and "there is a barber") is indeed a paradox)*

Exercise 31: (Runtime)
Apply the resolution calculus to

$$F = \forall x\ (P(x) \vee Q(x)) \wedge \forall y(\neg Q(y) \vee Q(f(y))).$$

After how many resolution steps does it terminate? Justify your answer.
(„one resolution" step means adding one resolvent.)

Exercise 32: (Find the box)
Use the resolution calculus to show that $\square \in \mathrm{Res}(F)$ for

$$F \equiv \big\{\{\neg Q(h(b), y), R(z, a)\}, \{P(a, y, f(h(y))), R(y, a), Q(h(b), b)\}, \{\neg P(a, z, f(h(b))), R(x, a)\}, \{\neg R(y, a)\}\big\}$$

Hand in your solutions until 11.12.2018 at 14:00 in post box 2183 in V3,
or via email to the tutor.

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