Dr. D. Frettlöh 18.12.2018

Formal Logic — Exercise Sheet 10

Exercise 37: (Undecidable problem I)
For each of the following instances of the Post correspondence problem, find a solution for it or show that it has no solution.

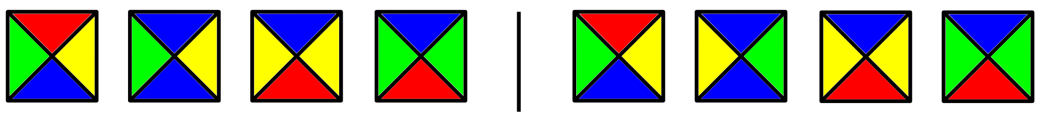
(a) $a_1 = 010, a_2 = 0, a_3 = 11$ and $b_1 = 0, b_2 = 10, b_3 = 01$.

(b) $a_1 = 101, a_2 = 0, a_3 = 01$ and $b_1 = 10, b_2 = 01, b_3 = 010$.

(c) $a_1 = 1, a_2 = 0, a_3 = 010, a_4 = 11$ and $b_1 = 10, b_2 = 10, b_3 = 01, b_4 = 1$.

(d) $a_1 = 10, a_2 = 011, a_3 = 101$ and $b_1 = 101, b_2 = 11, b_3 = 011$.

Exercise 38: (Undecidable problem II)
For each of the two collections of four Wang tiles below, prove that they can tile the plane (according to the rules: squares are placed vertex-to-vertex, adjacent edges carry the same colour, tiles are not rotated or reflected), or show that there is no such tiling.



Exercise 39: (Undecidable problem III: Mortal Matrices)
The mortal matrix problem asks for a given set of matrices $A_1, \ldots, A_m \in \mathbb{R}^{n\times n}$ whether there is a way to multiply the matrices such that the result is the matrix containing zeros only. It is allowed to use the same matrix more than once. I.e., we are looking for a sequence $i_1, i_2, \ldots, i_k$ with $1 \le i_j \le m$ such that

$$A_{i_1} \cdot A_{i_2} \cdots A_{i_k} = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{pmatrix}.$$

If this is possible the set $\{A_1, \ldots, A_m\}$ is called *set of mortal matrices*. Decide which of the following sets of matrices are sets of mortal matrices. Give either an example for a product yielding the zero matrix, or give a convincing reason why this is not possible.

(a) $A_1 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, $A_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $A_3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

(b) $B_1 = \begin{pmatrix} 1 & -2 \\ -1 & 2 \end{pmatrix}$, $B_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $B_3 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

(c) $C_1 = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$, $C_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $C_3 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

Exercise 40: (Post correspondence problem light)
Show that the following variant of the Post correspondence problem is decidable.

Given two finite lists $u_1, \ldots, u_n$ and $v_1, \ldots, v_n$ of finite words in $\{0,1\}^* \setminus \{\varepsilon\}$, where ε denotes the empty word with no letters. A solution to this problem is a sequence of indices $i_1, i_2, \ldots, i_k$, $j_1, j_2, \ldots, j_m$ with $k \ge 1$ such that

$$u_{i_1} \ldots u_{i_k} = v_{j_1} \ldots v_{j_m}.$$

Hand in your solutions until 8.1.2018 at 14:00 in post box 2183 in V3,
or via email to the tutor.

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