Dr. D. Frettlöh 8.1.2019

Formal Logic — Exercise Sheet 11

Exercise 41: (Football experts)
Formulate the following statements as formulas in modal logic.

(a) "Werder Bremen will win all remaining matches"

(b) "Borussia Dortmund will not win all remaining matches"

(c) "If Bayern München will not win its match on some weekend then Werder Bremen will not win at the same weekend"

(d) "If Bayern München wins its match then Borussia Dortmund wins its match on the same weekend"

(e) "On each weekend at least one of Bayern München, Borussia Dortmund and Werder Bremen will win"

Moreover, show that at least one of these statements is wrong.

Exercise 42: (Frames)
Given the frame $W = \mathbb{N}$, let R be the relation $<$ (hence $R = \{(n, m) \mid n, m \in \mathbb{N}, n < m\}$), and the valuation $\alpha : \{A, B\} \times \mathbb{N} \to \{0, 1\}$ be given by

$$\alpha(A, n) = \begin{cases} 1 & \text{if } n \text{ odd} \\ 0 & \text{else} \end{cases} \qquad \alpha(B, n) = \begin{cases} 1 & \text{if } n < 7 \\ 0 & \text{else} \end{cases}$$

Determine the truth values of the following formulas in the point $s = 4$.

(a) $F = \diamond \diamond \diamond B$

(b) $G = \diamond \Box \neg B$

(c) $H = \diamond(A \wedge \Box \neg B)$

Exercise 43: (Relations and directed graphs)
Visualise the following relations as directed graphs $G = (W, R)$. I.e., the nodes of G are the elements of W, the edges of G are the (ordered!) elements of R. Visualise also (W, R^2) and (W, R^3) in each case.

(a) $W = \{1, 2, 3, 4, 5\}$, $R = \{(n, m) \mid n, m \in W, |n - m| \leq 1\}$

(b) $W = \{0, 1, 2, 3, 4, 5\}$, $R = \{(n, m) \mid n, m \in W, n + m = 0 \bmod 3\}$

(c) $W = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$, $R = \{(n, m) \mid n, m \in W, n \subseteq m\}$

State for each of the nine relations (W, R^i) $(i = 1, 2, 3)$ whether they are reflexive, and/or symmetric, and/or transitive.

Exercise 44: (More rules)
(a) Prove Rule 5 of Theorem 4.2 (you may use rules 1,2,6 and 7).
(b) Prove Rule 4 of Theorem 4.2 by showing that $\Box(F \Rightarrow G) \Rightarrow (\diamond F \Rightarrow \diamond G)$ is a tautology. (You may use rules 1,2,6 and 7.)
(c) Show that the converse of Rule 4, i.e. $(\diamond F \Rightarrow \diamond G) \Rightarrow \Box(F \Rightarrow G)$, does not hold (for instance by providing a counterexample).

Hand in your solutions until 15.1.2019 at 14:00 in post box 2183 in V3,
or via email to the tutor.

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