Dr. D. Frettlöh 8.10.2019

Formal Logic — Exercise Sheet 1

Exercise 1: (Satisfiable, tautology, equivalent)
Consider the formulas

$$F_1 = (A \wedge \neg B) \vee C, \quad F_2 = A \Rightarrow (B \Rightarrow C), \quad F_3 = (A \Rightarrow B) \Rightarrow C$$

Decide for each of them: is the formula a tautology? Is the formula satisfiable? If it is, give a satisfying valuation $\mathcal{A}$ for the respective formula. Are F_1 and F_2 equivalent? Are F_1 and F_3 equivalent? Are F_2 and F_3 equivalent?

Exercise 2: (Disney^{TM} Princesses^{TM})
(a) The three Disney^{TM} princesses^{TM} Arielle^{TM}, Belle^{TM} and Cinderella^{TM} are invited to a party. They make the following statements:

Arielle^{TM}:	I will come to the party only if Belle^{TM} and Cinderella^{TM} will come as well; otherwise I will not come.
Belle^{TM}:	If Cinderella^{TM} is coming I will not come.
Cinderella^{TM}:	If Arielle^{TM} will not come I will come to the party.

Translate the statements into a formula F in propositional logic. Is F satisfiable? If so, give a valuation $\mathcal{A}$ such that $\mathcal{A} \models F$.

(b) Do the same for the following statements.

Arielle^{TM}:	I will come to the party only if Belle^{TM} or Cinderella^{TM} (or both) will come as well; otherwise I will not come.
Belle^{TM}:	I will come to the party only if Arielle^{TM} will not come, and I will not come to the party if Arielle^{TM} will come.
Cinderella^{TM}:	I will not come to the party.

Exercise 3: (exclusive or)
The "exclusive or" $A \oplus B$ is defined via $\mathcal{A}(A \oplus B) = 1$ if and only if $\mathcal{A}(A) \neq \mathcal{A}(B)$. (This is the same as $\neg(A \Leftrightarrow B)$). Find a formula including $\oplus$ that is a tautology; and find a formula including $\oplus$ that is unsatisfiable.

Exercise 4: (Minimal sets of operators)
(a) Prove that for each formula F there is an equivalent formula G using only $\neg$ and $\wedge$.

(b) Prove that there is some formula F such that there is no formula G equivalent to F, where G uses only $\vee$ and $\wedge$.

(c) Prove that for each formula F there is an equivalent formula G using only NAND (meaning "not and"), where the NAND operator $\uparrow$ is defined by $A \uparrow B := \neg(A \wedge B)$.

Hand in your solutions until Mon 14.10.2019 at 11:00 in post box 2183 in V3,
or via email to the respective tutor.

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