

Formal Logic — Exercise Sheet 2**Exercise 5: (Laws of Logic)**

(a) Prove the following laws using truth tables, compare Theorem 1.1.

1. $F \wedge (F \vee G) \equiv F, F \vee (F \wedge G) \equiv F$ (Absorption)
2. $\neg(F \wedge G) \equiv \neg F \vee \neg G$ (de Morgan's law 1)

(b) Show the following equivalences using the laws in Theorem 1.1 and the definitions of \Leftrightarrow and \Rightarrow only (compare Example 1.4). For each transformation state the law you used.

1. $\neg(F \vee G \vee H) \equiv \neg F \wedge \neg G \wedge \neg H$
2. $F \Leftrightarrow G \equiv (F \Rightarrow G) \wedge (G \Rightarrow F)$
3. $F \wedge (G \Leftrightarrow H) \equiv (F \wedge G \wedge H) \vee \neg(\neg F \vee G \vee H)$

Exercise 6: (CNF and DNF)

Transform the following formulas into conjunctive normal form and into disjunctive normal form, using Algorithm 1.1 shown in the lecture.

$$F = \neg\left(A \vee \neg(B \wedge \neg(C \vee D))\right) \wedge (A \Rightarrow B), \quad G = (A \wedge B \wedge \neg C) \vee (D \wedge \neg E)$$

Exercise 7: (Borrowean formulas)

(a) Find three formulas F_1, F_2, F_3 such that $F_i \wedge F_j$ is satisfiable for all choices of $1 \leq i < j \leq 3$, but $F_1 \wedge F_2 \wedge F_3$ is not satisfiable.

(b) Find four formulas F_0, F_1, F_2, F_3 such that $F_i \wedge F_j \wedge F_k$ is satisfiable for all choices of $i, j, k \in \{0, 1, 2, 3\}$, but $F_0 \wedge F_1 \wedge F_2 \wedge F_3$ is not satisfiable.

Exercise 8: (Switch and and or)

Let $F \equiv G$. Let neither F nor G contain any \Leftrightarrow or \Rightarrow . Let F' (respectively G') be the resulting formulas if one changes each \vee in F (respectively G) into \wedge and vice versa. Prove that $F' \equiv G'$.

Hand in your solutions until 21.10.2019 at 11:00 in post box 2183 in V3,
or via email to the tutor.

Please indicate the name of the tutor on your solution sheet.

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