Formal Logic   —   Exercise Sheet 2

Exercise 5: (Laws of Logic)
(a) Prove the following laws using truth tables, compare Theorem 1.1.

1. $F \wedge (F \vee G) \equiv F$, $F \vee (F \wedge G) \equiv F$ (Absorption)

2. $\neg(F \wedge G) \equiv \neg F \vee \neg G$ (de Morgan's law 1)

(b) Show the following equivalnces using the laws in Theorem 1.1 and the definitions of $\Leftrightarrow$ and $\Rightarrow$ only (compare Example 1.4). For each transformation state the law you used.

1. $\neg(F \vee G \vee H) \equiv \neg F \wedge \neg G \wedge \neg H$

2. $F \Leftrightarrow G \equiv (F \Rightarrow G) \wedge (G \Rightarrow F)$

3. $F \wedge (G \Leftrightarrow H) \equiv (F \wedge G \wedge H) \vee \neg(\neg F \vee G \vee H)$

Exercise 6: (CNF and DNF)
Transform the following formulas into conjunctive normal form and into disjunctive normal form, using Algorithm 1.1 shown in the lecture.

$$F = \neg\Big(A \vee \neg\big(B \wedge \neg(C \vee D)\big)\Big) \wedge (A \Rightarrow B), \qquad G = (A \wedge B \wedge \neg C) \vee (D \wedge \neg E)$$

Exercise 7: (Borromean formulas)
(a) Find three formulas F_1, F_2, F_3 such that $F_i \wedge F_j$ is satisfiable for all choices of $1 \leq i < j \leq 3$, but $F_1 \wedge F_2 \wedge F_3$ is not satisfiable.

(b) Find four formulas F_0, F_1, F_2, F_3 such that $F_i \wedge F_j \wedge F_k$ is satisfiable for all choices of $i, j, k \in \{0, 1, 2, 3\}$, but $F_0 \wedge F_1 \wedge F_2 \wedge F_3$ is not satisfiable.

Exercise 8: (Switch and and or)
Let $F \equiv G$. Let neither F nor G contain any $\Leftrightarrow$ or $\Rightarrow$. Let F' (respectively G') be the resulting formulas if one changes each $\vee$ in F (respectively G) into $\wedge$ and vice versa. Prove that $F' \equiv G'$.