Dr. D. Frettlöh 29.10.2019

# Formal Logic — Exercise Sheet 3

**Exercise 9: (Horn formula algorithm)**
**(a)** Apply the Marking Algorithm for Horn formulas to the following two formulas $F$ and $G$. Is $F$ (resp. $G$) satisfiable? If yes, please give all valuations $\mathcal{A}$ with $\mathcal{A} \models F$ (resp. $\mathcal{A} \models G$).

$$F = (D \wedge A \Rightarrow B) \wedge (B \wedge E \Rightarrow C) \wedge (C \Rightarrow D) \wedge (D \wedge C \Rightarrow A) \wedge (1 \Rightarrow C)$$

$$G = (\neg A_1 \vee \neg A_3 \vee \neg A_5 \vee A_6) \wedge (\neg A_2 \vee \neg A_3 \vee A_6) \wedge \neg A_7 \wedge (\neg A_6 \vee \neg A_2 \vee A_7) \wedge A_4 \wedge (A_5 \vee \neg A_4) \wedge (\neg A_4 \vee \neg A_5 \vee A_1) \wedge (\neg A_1 \vee \neg A_2 \vee A_3) \wedge (\neg A_5 \vee \neg A_1 \vee A_2)$$

**(b)** The five™ Disney™ princesses™ Arielle™, Belle™, Cinderella™, Diana™ and Eliza™ are invited to a party. Again they state strict opinions:

Arielle™: If Eliza™ and Belle™ are coming to the party I will come, too.
Belle™: If Eliza™ is coming I will come as well.
Cinderella™: If Arielle™ and Belle™ are coming I will come, too.
Diana™: If Eliza™ and Cinderella™ will come I will come, too.
Eliza™: I will go to the party anyway.

Translate their statements into a single Horn™ formula $F$. Is $F$ satisfiable? If yes, please give a valuation $\mathcal{A}$ with $\mathcal{A} \models F$. What is the minimal satisfying valuation?

**Exercise 10: (Easy decisions)**
**(a)** Show that any Horn formula (in CNF) is satisfiable if each disjunctive clause contains at least one $\neg$.

**(b)** Give an algorithm that decides in polynomial time whether a formula in disjunctive normal form (DNF) is satisfiable.

**Exercise 11: (satisfiable vs tautology)**
Prove or give a counterexample:

**(a)** If $F$ is a tautology and $F \Rightarrow G$ is a tautology, then $G$ is a tautology.

**(b)** If $F$ is satisfiable and $F \Rightarrow G$ is satisfiable, then $G$ is satisfiable.

**(c)** If $F$ is satisfiable and $F \Rightarrow G$ is a tautology, then $G$ is satisfiable.

**(d)** If $F$ is satisfiable and $F \Rightarrow G$ is a tautology, then $G$ is a tautology.

**Exercise 12: (Infinitely many formulas)**
Find all valuations for $A_1, A_2, \ldots$ satisfying the infinite set of formulas

$$\{A_1 \vee A_2, \neg A_2 \vee \neg A_3, A_3 \vee A_4, \neg A_4 \vee \neg A_5, A_5 \vee A_6, \ldots\}$$

*(Hint: there are more than seven.)*

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Hand in your solutions until 4.11.2019 at 11:00 in post box 2183 in V3,
or via email to the tutor.
Please indicate the name of the tutor on your solution sheet.


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