Formal Logic — Exercise Sheet 4

Exercise 13: (Resolvents)
Construct the CNF of the following formula F and determine $Res^1(F)$ and $Res^2(F)$. Is $\square \in Res^3(F)$? Is F satisfiable?

$$F = (A \vee \neg(B \wedge \neg C)) \wedge (B \vee C) \wedge (A \Rightarrow C) \wedge (C \Rightarrow B) \wedge \neg C.$$

Exercise 14: (Modus tollens and resolution are consequences)
(a) Prove the modus tollens. That is, show that $\{F \Rightarrow G, \neg G\} \vdash \neg F$, if F and G are two formulas in propositional logic.

(b) Prove Lemma 1.9. That is, show that $\{F \vee L, G \vee \neg L\} \vdash F \vee G$, if F and G are formulas and L is some literal.

Exercise 15: (The buying public)
Family Smith plans to buy a car, a moped and a washing machine next year. However, if Mrs Smith fails to receive an incentive on top of her salary the Smiths cannot afford all three of them. The washing machine will be bought in any case. They need at least one motor vehicle, too. If they spend their holiday in Spain they cannot afford the car. If they do not spend their holiday in Spain they need to buy the moped in order to conciliate their spoilt son who is slightly mentally unstable.

Model the situation into a formula in propositional logic. Show by using the resolution method that family Smith will buy a moped and not a car if Mrs Smith fails to receive her inventive.

Exercise 16: (Testing tautology and consequence via resolution)
(a) Using resolution, show that

$$F = (\neg B \wedge \neg C \wedge D) \vee (\neg B \wedge \neg D) \vee (C \wedge D) \vee B$$

is a tautology.

(b) Using resolution, show that $A \wedge B \wedge C$ is a consequence of the set of formulas

$$F = \{\neg A \vee B, \neg B \vee C, A \vee \neg C, A \vee B \vee C\}.$$

Hand in your solutions until 11.11.2019 at 11:00 in post box 2183 in V3,
or via email to the tutor.

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