Dr. D. Frettlöh 12.11.2019

Formal Logic — Exercise Sheet 5

Exercise 17: (Variations of König's Lemma)

(a) Show that König's Lemma is wrong if we do not require that each vertex has finitely many neighbours, even if each level of the tree is not empty (I.e., for each $n \in \mathbb{N}$ there is a vertex in the tree with distance n from the root). Hence: construct a counterexample, that is, find a tree T that has infinitely many vertices, each level of T is not empty, and T does not contain any infinite path.

(b) Let T be a tree such that each vertex has only finitely many neighbours, and such that T does not contain any infinite path. Show that T has only finitely many vertices.

Exercise 18: (Testing another tautology via resolution)

Using resolution, show that

$$F = (A \wedge B \wedge D) \vee (\neg B \wedge C \wedge D) \vee (B \wedge \neg A) \vee \neg D \vee (\neg B \wedge \neg C \wedge D)$$

is a tautology.

Exercise 19: (Resolution variants)

(a) Can the resolution of two clauses in Horn formulas be a clause that is not a Horn formula? If "yes" provide an example. If "no" give a convincing reason why not.

(b) Is the resolution calculus still correct if we allow for the simultaneous resolution of two literals? That means for instance $\big\{\{A, B, C\}, \{\neg A, \neg B, C\}\big\} \to \{C\}$. If "yes", give a convincing reason why. If "no" give a counterexample.

Exercise 20: (Efficiency of resolution calculus)

(a) Let F be a formula in CNF using n different atomic formulas. Show that in the resolution algorithm the number of steps is at most $O(n^2)$ if all clauses in the CNF of F have length at most two. Here "one step" is taking one resolvent.
(This shows essentially that the runtime for 2SAT is $O(n^2)$, i.e., 2SAT is efficiently decidable.)

(b) Let F be a clause set with m clauses containing the atomic formulas $A_1, \dots, A_n$. Find (and prove) an upper bound for the maximal value of $|Res^*(F)|$? (Here $|M|$ denotes the number of elements of a set M.)

(c) Prove that there is $k \in \mathbb{N}$ such that

$$Res^k(F) = Res^{k+1}(F) = \cdots = Res^*(F).$$

(This shows that the resolution calculus always terminates for formulas in propositional logic.)

Hand in your solutions until 18.11.2019 at 11:00 in post box 2183 in V3,
or via email to your tutor.

Tutors:	Oliver Tautz	otautz@techfak.uni-bielefeld.de	Wed 8-10
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