Formal Logic   —   Exercise Sheet 6

Exercise 21: (Dr Who pressing buttons)
Dr Who encounters some device about to explode, thus destroying all life, time and space existing. It has four buttons: A, B, C and D. The device can be shut down (hence not explode) by pressing the correct button. It is known that if button A is pressed then button B must be pressed, too. If button C is pressed then button D must be pressed, too. If button D is pressed then either button A or button B (or both) must be pressed, too. Moreover, pressing neither button A nor button B leads to immediate disaster.

Translate the situation above into a formula in propositional logic and apply the tableau calculus in order to (a) decide whether Dr Who is able to save the universe (i.e., whether the formula is satisfiable), and (b) find the correct button.

Exercise 22: (Meaning of closed paths)
(a) If the tableau calculus applied to F yields a tableau with no closed path, is F a tautology? Justify your answer.

(b) If the tableau calculus appled to $\neg F$ yields a tableau with closed paths only, is F a tautology? Justify your answer.

Exercise 23: (Relations)
Which of the following relations are equivalence relations (on the corresponding sets)? Give a convincing reasion why they are, or provide a counterexample. Let B denote the set of all people living in Bielefeld. For the ones that are equivalence relations: list all equivalence classes.

1. $R = \varnothing$ on B.
2. $R = B \times B$ on B.
3. $R = \{(a,b) \mid a, b \in \mathbb{Z}, a \le b\})$ on $\mathbb{Z}$.
4. $R = \{(a,b) \mid a, b \in \mathbb{Z}, |a - b| \le 2\})$ on $\mathbb{Z}$.
5. $R = \{(a,b) \mid a, b \in \mathbb{Z}, \exists k \in \mathbb{Z} : a - b = 5k\})$ on $\mathbb{Z}$.
6. $R = \{(a,b) \mid a, b \in B, a \text{ and } b \text{ have the same birthday}\}$.

Exercise 24: (Structures and models)
List all partial formulas of the following formula, and all terms, and write down the matrix of F.

$$F = \exists x\, \forall y\, \big(P(x, y, f(x)) \wedge Q(x, y)\big)$$

Find two structure that are not models for F, and two structures that are models for F.

Hand in your solutions until 25.11.2019 at 11:00 in post box 2183 in V3,
or via email to your tutor.

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