Formal Logic — Exercise Sheet 7

Exercise 25: (Reflexive, symmetric, transitive)
Consider the formulas

$$F_1 = \forall x\, P(x,x), \quad F_2 = \forall x\, \forall y\, \big(P(x,y) \Rightarrow P(y,x)\big), \quad F_3 = \forall x\, \forall y\, \forall z\, \big((P(x,y) \wedge P(y,z)) \Rightarrow P(x,z)\big).$$

Show that none of these formulas is a consequence of the other two by constructing structures where F_1 and F_2 are true, but F_3 is not; respectively where F_1 and F_3 are true, but F_2 is not; respectively where F_2 and F_3 are true, but F_1 is not.

Exercise 26: (Small universes)
(a) Find a formula F of first-order logic without free variables such that F is satisfiable only if $U_{\mathcal{A}}$ has at least three elements. (I.e., F is unsatisfiable for all $U_{\mathcal{A}}$ where $U_{\mathcal{A}}$ has only one or two elements.)

(b) Find a formula F of first-order logic with identity (see Remarks 2.1 and 2.4), and without free variables, such that for all $\mathcal{A}$ with $\mathcal{A} \models F$ holds that $U_{\mathcal{A}}$ has at most two elements.

Exercise 27: (Big universes)
Let F be a formula and $\mathcal{A}$ a model for F with $|U_{\mathcal{A}}| = n$. (The notation $|M|$ means the number of elements of some set M.) For each $m > n$ construct a model $\mathcal{A}_m$ for F with $|U_{\mathcal{A}_m}| = m$. Construct a further model $\mathcal{A}'$ for F such that $U_{\mathcal{A}'}$ has infinitely many elements.

This exercise seems to contradict 26 (b). Explain why this is not a contradiction.

Exercise 28: (Not a law)
Show that the two formulas in (a) (respectively, in (b)) are not equivalent to each other by providing a (counter-) example for each.

(a) $(\forall x\, F) \vee (\forall x\, G) \quad \not\equiv \quad \forall x(F \vee G)$,

(b) $(\exists x\, F) \wedge (\exists x\, G) \quad \not\equiv \quad \exists x(F \wedge G)$.

Hand in your solutions until 2.12.2019 at 11:00 in post box 2183 in V3,
or via email to your tutor.

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