Dr. D. Frettlöh 3.12.2019

Formal Logic — Exercise Sheet 8

Exercise 29: (Drinker paradox)
In each (nonempty) pub there is someone such that, if this person is drinking, then everyone in the pub is drinking.

Translate the statement above into a formula F in first-order logic. Prove that it is valid by showing that $\neg F$ is unsatisfiable. That is, establish the Skolem normal form of $\neg F$ and apply the resolution calculus to it.
(Hence this is not really a paradox, it only sounds like one.)

Exercise 30: (I can't get no satisfaction)
Establish the Skolem normal form of

$$F = \forall x \, \big(P(x) \vee Q(x)\big) \wedge \forall x \big(\neg Q(x) \vee Q(f(x))\big)$$

and apply the resolution calculus. Is F satisfiable?

Exercise 30: (Barber paradox)
In a remote small town there are two strict rules for barbers:

1. Each barber shaves all those who do not shave themselves.

2. No barber shaves someone who shaves himself.

Show that there is no barber in this town, by translating 1. and 2. and "there is a barber" into formulas F, G and H in first-order logic and use the resolution calculus in order to show that $F \wedge G \wedge H$ is unsatisfiable. *(Hint: use one predicate for "x is barber" and another one for "x shaves y". This one is indeed a paradox.)*

Exercise 32: (Infinite models)
Prove that there are formulas having infinite models only. That is:

(a) Show that $F = \forall x \; P(x, f(x)) \wedge \forall y \; \neg P(y,y) \wedge \forall x \; \forall y \; \forall z \; \big((P(x,y) \wedge P(y,z) \Rightarrow P(x,z)\big)$ is satisfiable by finding a model for F.

(b) Show that F is not satisfiable for any $\mathcal{A} = (U_{\mathcal{A}}, I_{\mathcal{A}})$ where $U_{\mathcal{A}}$ is finite.

Hand in your solutions until 9.12.2019 at 11:00 in post box 2183 in V3,
or via email to your tutor.

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