Formal Logic — Exercise Sheet 9

Exercise 33: (Guess the cardinality)
Determine the cardinality of each of the following sets. You do not need to justify your answer.

(a) $\mathbb{Q}^2$
(b) $\mathbb{R}^2$
(c) $\mathcal{P}(\mathcal{P}(\mathbb{R}))$
(d) $H(F)$ (Herbrand universe of some formula in first-order logic)
(e) the set $F := \{f : \mathbb{R} \to \mathbb{R}\}$ of all functions from $\mathbb{R}$ to $\mathbb{R}$
(f) $\mathcal{P}(F)$

Exercise 34: (Prove the cardinality)
Give the cardinality of each of the following sets and prove your answer.

(a) $\mathcal{P}_{fin}(\mathbb{N})$.

(b) The set of all infinite 0-1-words: $\{u_1u_2, \ldots \mid u_i \in \{0,1\}\}$.

(c) the set of all finite 0-1-words: $\{u_1u_2 \ldots u_n \mid u_i \in \{0,1\}, n \in \mathbb{N}_0\}$.

(d) The set of all different Turing machines.

(e) The set of all sequences $(a_n)_{n\in\mathbb{N}}$ with values $a_n \in \mathbb{R}$.

The term in (a) denotes all finite subsets of $\mathbb{N}$, i.e. $\mathcal{P}_{fin}(\mathbb{N}) := \{M \subset \mathbb{N} \mid M \text{ finite }\}$. In (b) the set contains all infinite words (strings) made from the letters 0 and 1, for instance 010101010...., 101000111010..., 00000000... and so on. In (c) the set contains all finite words with letters 0 or 1, i.e.: 0, 1, 00, 01, 10, 11, 000, 001, 010, ...; including the empty word ϵ with zero letters.

Exercise 35: (Tricky bijections)
(a) Find a bijection f from the closed interval $[0,1]$ into the open interval $]0,1[$.

(b) Find a bijection f from $\mathbb{Q}^2$ to $\mathbb{Q}$.

(Hint: You may describe the bijection in any way, e.g., using `if...then`*, or words, or combinations of other bijections)*

Exercise 36: (Undecidable problems)
For each of the following instances of the Post correspondence problem, find a solution for it or show that it has no solution.

(a) $u_1 = 010, u_2 = 0, u_3 = 11$ and $v_1 = 0, v_2 = 10, v_3 = 01$.

(b) $u_1 = 010, u_2 = 0, u_3 = 11, u_4 = 10$ and $v_1 = 10, v_2 = 10, v_3 = 01, v_4 = 101$.

(c) $u_1 = 00, u_2 = 1, u_3 = 101, u_4 = 0$ and $v_1 = 0, v_2 = 01, v_3 = 10, v_4 = 01$.

(d) $u_1 = 01, u_2 = 100, u_3 = 010$ and $v_1 = 010, v_2 = 00, v_3 = 100$.

Hand in your solutions until 16.12.2019 at 11:00 in post box 2183 in V3,
or via email to your tutor.

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