DR. D. FRETTLÖH 17.12.2019

Formal Logic — Exercise Sheet 10

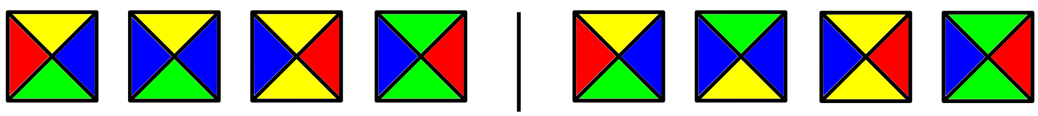
Exercise 37: (Undecidable problem II: Mortal Matrices)
Decide which of the following sets of matrices are sets of mortal matrices. Give either an example for a product yielding the zero matrix, or give a convincing reason why this is not possible.

(a) $A_1 = \left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix}\right)$, $A_2 = \left(\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}\right)$, $A_3 = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$.

(b) $B_1 = \left(\begin{smallmatrix} 1 & -1 \\ -2 & 2 \end{smallmatrix}\right)$, $B_2 = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$, $B_3 = \left(\begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix}\right)$.

(c) $C_1 = \left(\begin{smallmatrix} 1 & 2 \\ 1 & 2 \end{smallmatrix}\right)$, $C_2 = \left(\begin{smallmatrix} 1 & -1 \\ 1 & 0 \end{smallmatrix}\right)$, $C_3 = \left(\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}\right)$.

Exercise 38: (Undecidable problem III: Wang tiles)
For each of the two collections of four Wang tiles below, prove that they can tile the plane (according to the rules: squares are placed vertex-to-vertex, adjacent edges carry the same colour, tiles are not rotated or reflected), or show that there is no such tiling.



Exercise 39: (Decidable first-order logic)
Monadic first-order logic is first-order logic without function symbols, and where all predicates have only one input. Let F be a formula in monadic first-order logic containing n predicates $P_1, \ldots, P_n$.

(a) Show that if F is satisfiable, then there is a model $\mathcal{A} = (U_\mathcal{A}, I_\mathcal{A})$ for F such that $U_\mathcal{A}$ has 2^n elements. *(Hint: identify elements in $U_\mathcal{A}$ with respect to their values in $P_1^\mathcal{A}, \ldots, P_n^\mathcal{A}$.)*

(b) Use (a) to show that the problem "is a given formula F in monadic first-order logic satisfiable" is decidable.

Exercise 40: (Violate the Peano axioms!)
Peano axioms refers here to the Peano axioms in the lecture notes on page 37.

(a) Find a structure satisfying all Peano axioms except number 1.

(b) Find a structure satisfying all Peano axioms except number 6.

(c) Find a structure satisfying all Peano axioms except number 2.

You may tweak anything, e.g. $U_\mathcal{A} = \{0, 1, 2, 3\}$ rather than $U_\mathcal{A} = \mathbb{N}_0$, or $f(x, y) = 2x - y$ rather than $x + y$.

Hand in your solutions until 6.1.2020 at 11:00 in post box 2183 in V3,
or via email to your tutor.

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