

Formal Logic — Exercise Sheet 11**Exercise 41: (decidable vs undecidable vs semidecidable)**

Decide which of the following problems are decidable, which are undecidable but semidecidable and which are not even semidecidable. In each case give a short justification of your answer.

- (1) Travelling salesman problem, yes/no version: given a map and a number x , is there a tour of length $\leq x$?
- (2) Wang tile problem
- (3) The “halting-not-halting problem”: given two Turing machines t and t' and two inputs x and x' , decide whether t ever stops on input x and t' does never stop on input x' .
- (4) Determining whether a player has a winning strategy in the *Magic*TM cards game (given an arbitrary starting situation)
- (5) Determining whether a player has a winning strategy in chess (given an arbitrary starting situation)

Exercise 42: (Consequences of Peano axioms)

Show that the following statements are consequences of the Peano axioms. Note that there is no axiom stating $x + y = y + x$ (!) You may use the principle of induction.

- (a) $0 + x = x$
- (b) $S(x) + y = S(x + y)$
- (c) $y + x = x + y$

Exercise 43: (Football experts)

Formulate the following statements as formulas in modal logic. Moreover, show that the five formulas are not simultaneously satisfiable. That is, show that under each structure at least one of these statements is wrong.

- (a) “Werder Bremen will win all remaining matches”
- (b) “Borussia Dortmund will not win all remaining matches”
- (c) “If Bayern München will not win its match on some weekend then Werder Bremen will not win at the same weekend”
- (d) “If Bayern München wins its match then Borussia Dortmund wins its match at the same weekend”
- (e) “On each weekend at least one of Bayern München, Borussia Dortmund and Werder Bremen will win”

Exercise 44: (Frames)

Given the frame $W = \mathbb{N}$, let R be the relation $<$ (hence $R = \{(n, m) \mid n, m \in \mathbb{N}, n < m\}$), and the valuation $\alpha : \{A, B\} \times \mathbb{N} \rightarrow \{0, 1\}$ be given by

$$\alpha(A, n) = \begin{cases} 1 & \text{if } n \text{ odd} \\ 0 & \text{else} \end{cases} \quad \alpha(B, n) = \begin{cases} 1 & \text{if } n < 5 \\ 0 & \text{else} \end{cases}$$

Determine the truth values of the following formulas in the point $s = 3$.

- (a) $F = \diamond \diamond B$
- (b) $G = \diamond \Box \neg A$
- (c) $H = \diamond(A \wedge \Box \neg B)$

Hand in your solutions until 13.1.2020 at 11:00 in post box 2183 in V3,
or via email to your tutor.

Tutors: Oliver Tautz otautz@techfak.uni-bielefeld.de Wed 8-10
Jonas Kalinski jkalinski@techfak.uni-bielefeld.de Tue 16-18